Biological Polyelectrolytes and Biological Complexes

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DNA and Polyelectrolytes

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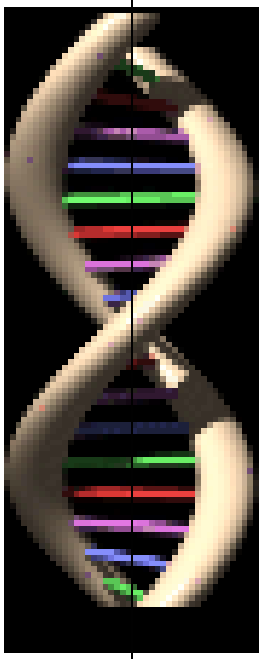
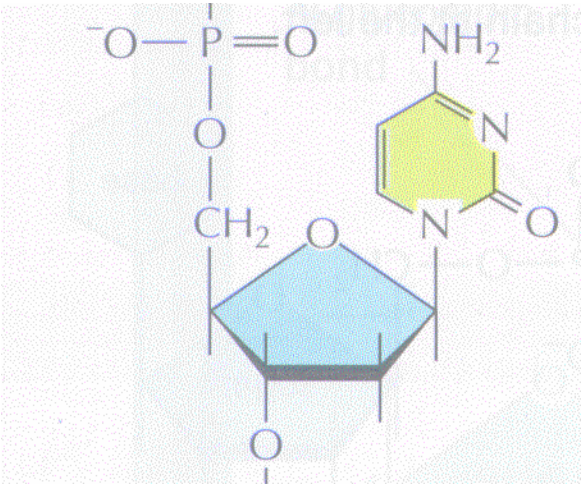


Unlike proteins, each of the DNA basic units (the base pair) is composed of two identical ionisable groups, the phosphate groups.



Projection of the two phosphates on the double-helix axis : (B-DNA)

Linear charge density : 2 e / 3.4 Å ~ 6 e / nm

equivalently, 1 e / 1.7 Å

→ One of the most highly charged systems

For proteins,

5 / 20 basic units may be ionized :

basic : lysine, arginine, histidine ($-NH_2^+$)

acidic : aspartic, glutamic ($-COO^-$)

Amino-acid size ~ 4 Å

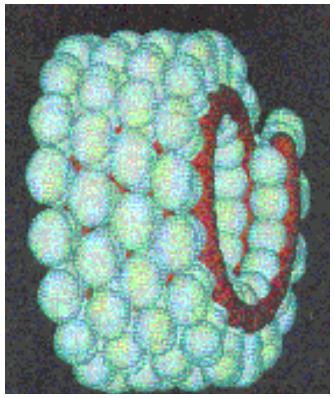
Typical Linear Charge density ~ $1 / (4 \times 4\ \text{Å}) \sim 0.6\ e / nm$

For the cell membrane,

Typical Surface Charge Density ~ $0.1\text{-}\ 1\ e / nm^2$

Proteins + (RNA or DNA) complexes :

Tobacco Mosaic Virus

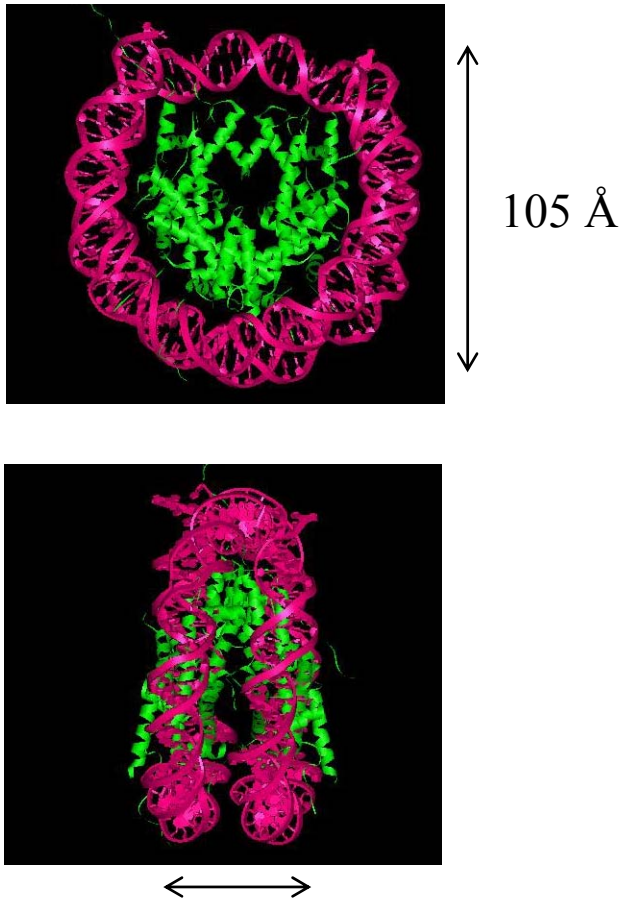


Linear Charge density

TMV ~ 2 e / nm

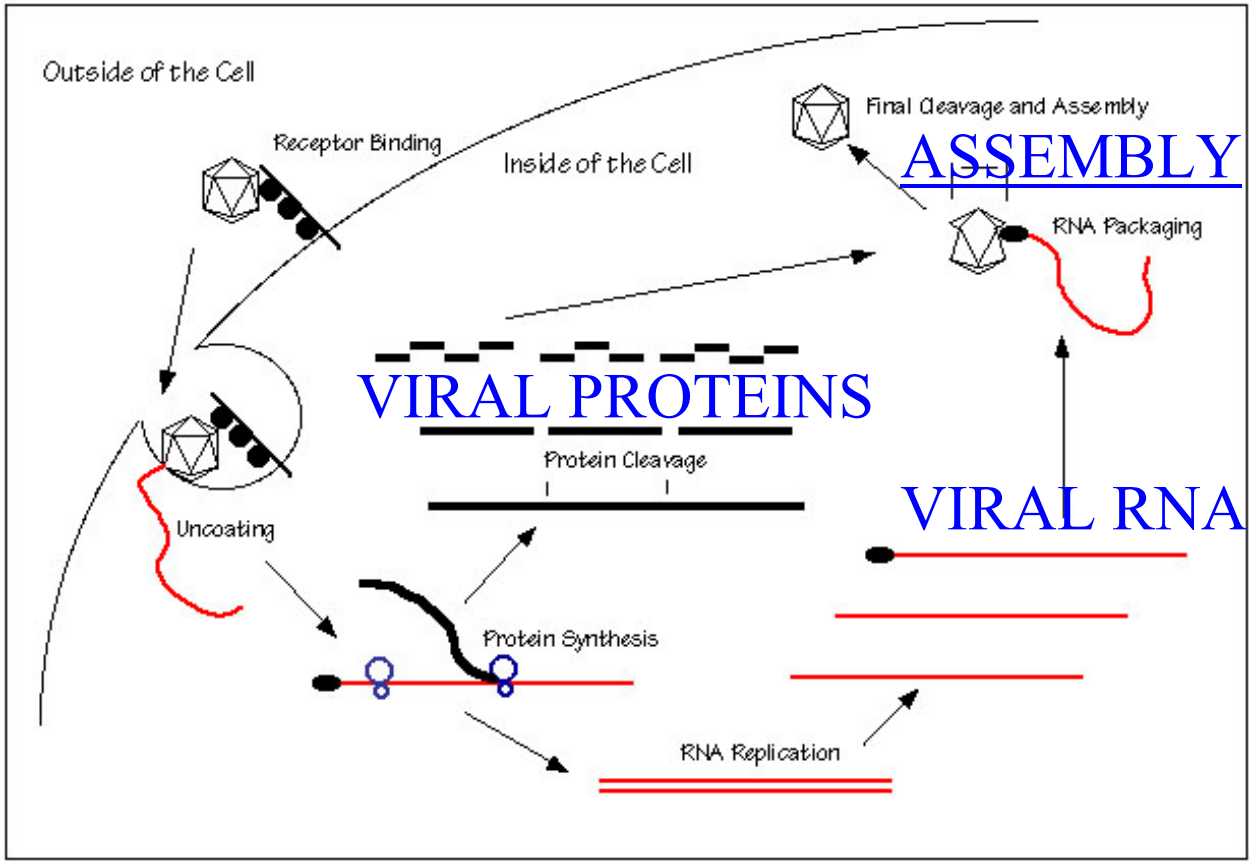
fd-virus ~ 5 e / nm

Nucleosome Core Particle



Surface Charge Density ~ 6 e / nm^2

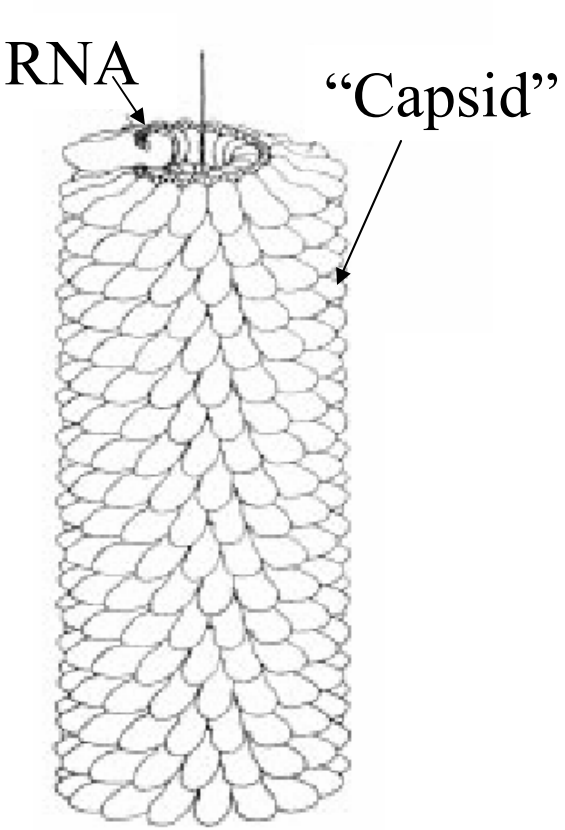
Virus Life Cycle (Polio)



TMV: one RNA of 6,300 base + 2,000 identical capsid proteins. RNA is a TEMPLATE

Viral Self-Assembly from solution of capsid protein + RNA molecules

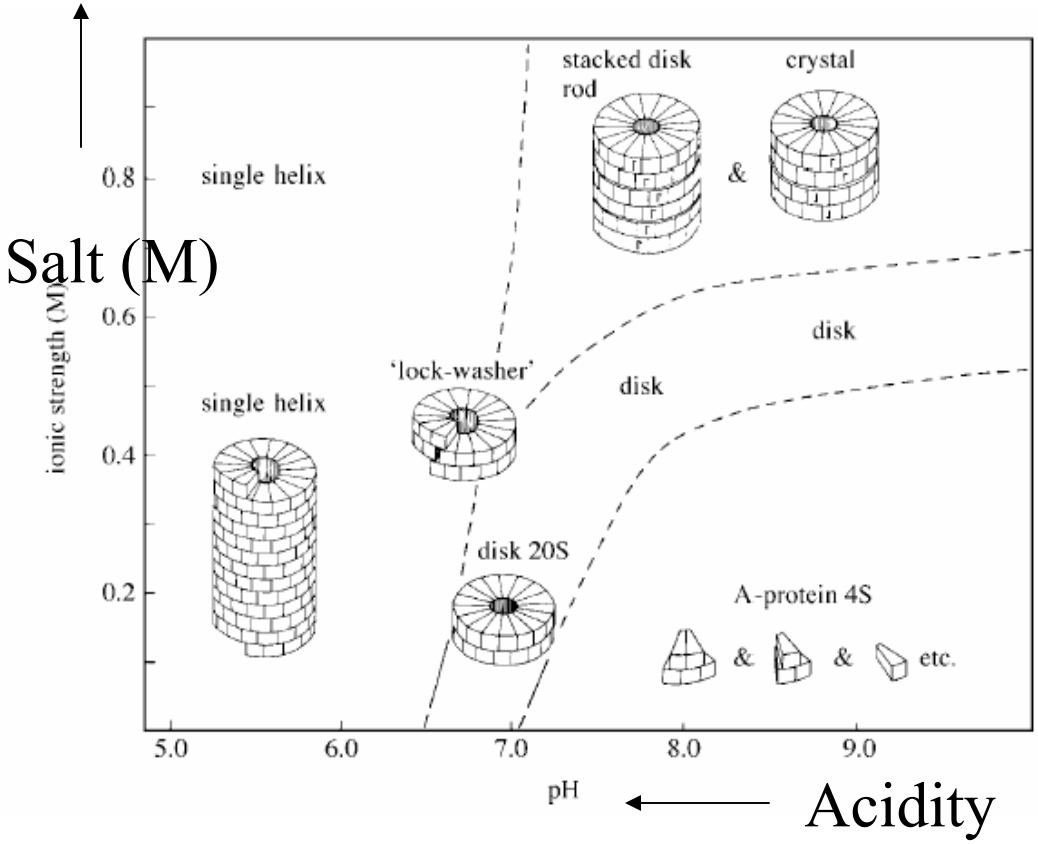
Tabaco Mosaic Virus



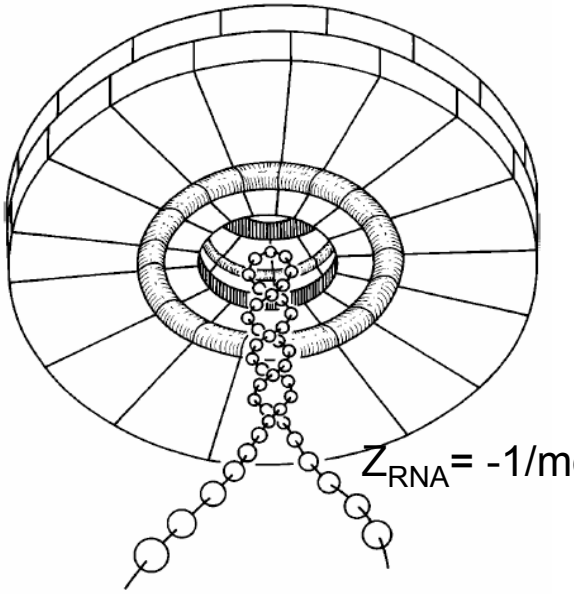
(Fraenkel-Conrat, 55)

1. Solution of TMV Capsid Proteins

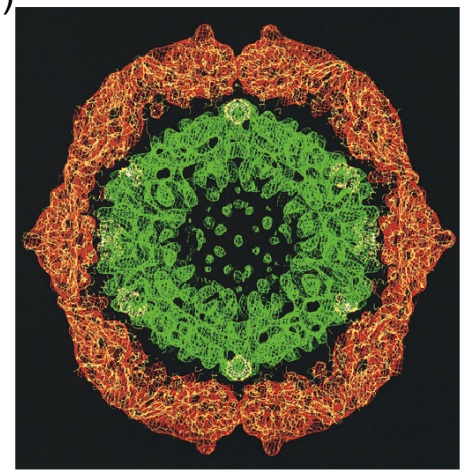
Electrostatic repulsion inhibits
+ protein +protein aggregation.



2. Add ssTMV+ RNA (assembly infective virus): Electrostatic attraction stabilizes viral assembly.

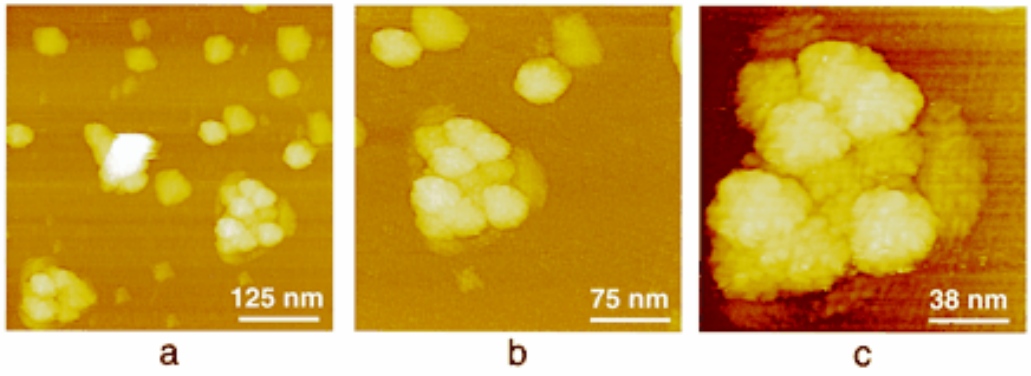


3. Sphere-like (icosahedral) RNA virus
(Tsuruta et al)



4. Capsid + Hole (density ≈ RNA crystal) self-assembles from capsid protein + viral RNA solution

Remove capsid proteins *except for charged amino acids*



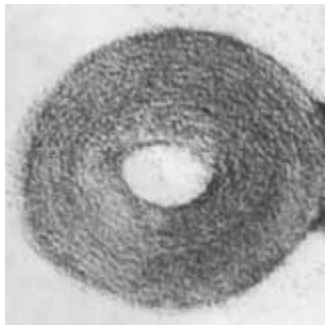
Geneo does not expands
"electrostatic virus"

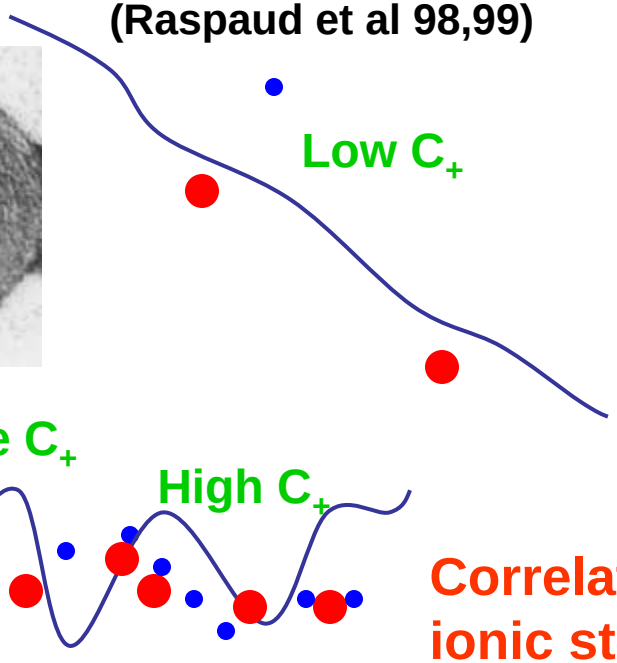
(A. McPherson)

Anionic and Cationic Complexes

DNA + cationic (4+) solutions

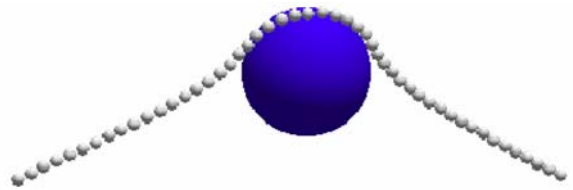
(Raspaud et al 98,99)



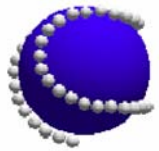


DNA – protein

(Netz, 04)



unwrapped (low/high salt)



wrapped
(intermediate salt)

Correlation (strong electrostatic coupling in ionic structures) + Counterion Release

Strong Polyelectrolyte Solution Properties

In Monovalent salts:

Water soluble

Stretched

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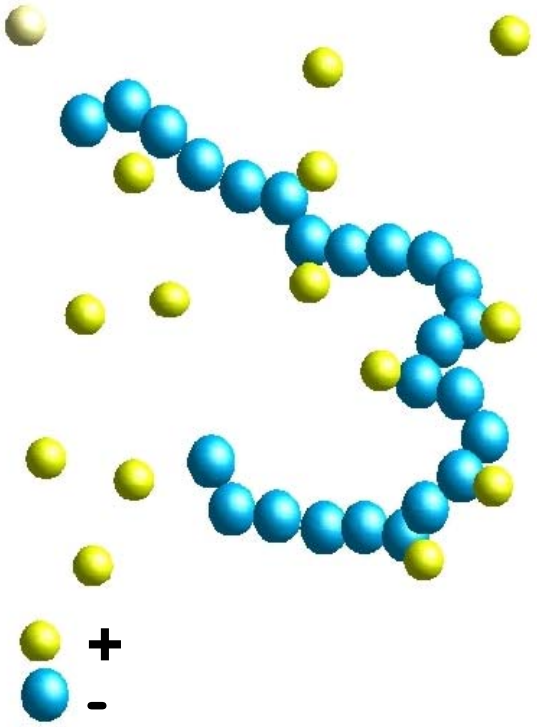
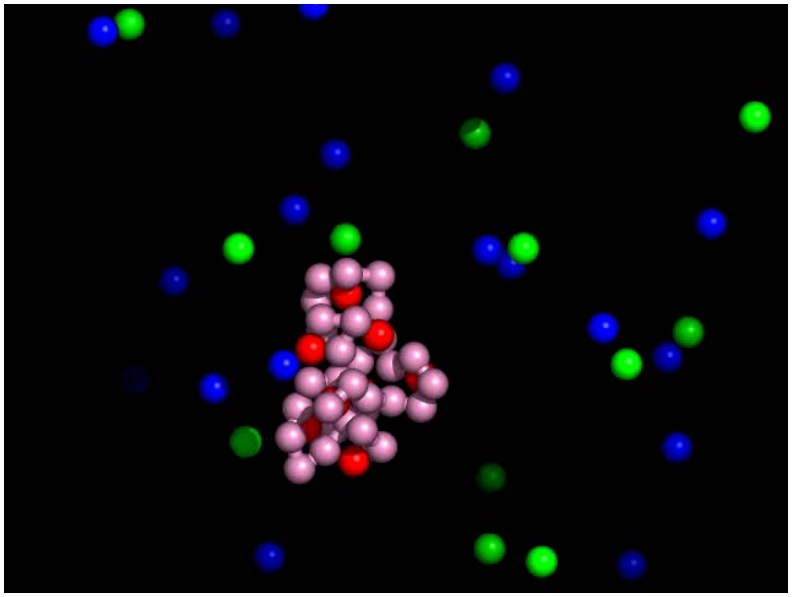


In Multivalent salts:

Precipitates

Collapsed (dense)



Olvera de la Cruz et al 1995

Gonzalez-Mozuelos et al 1995

Solis et al 2000, 2001

mer

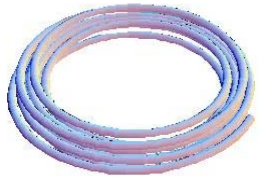
(+1)

(+4)

coion

(Luijten, 04)

Phase diagram: nearly universal



For

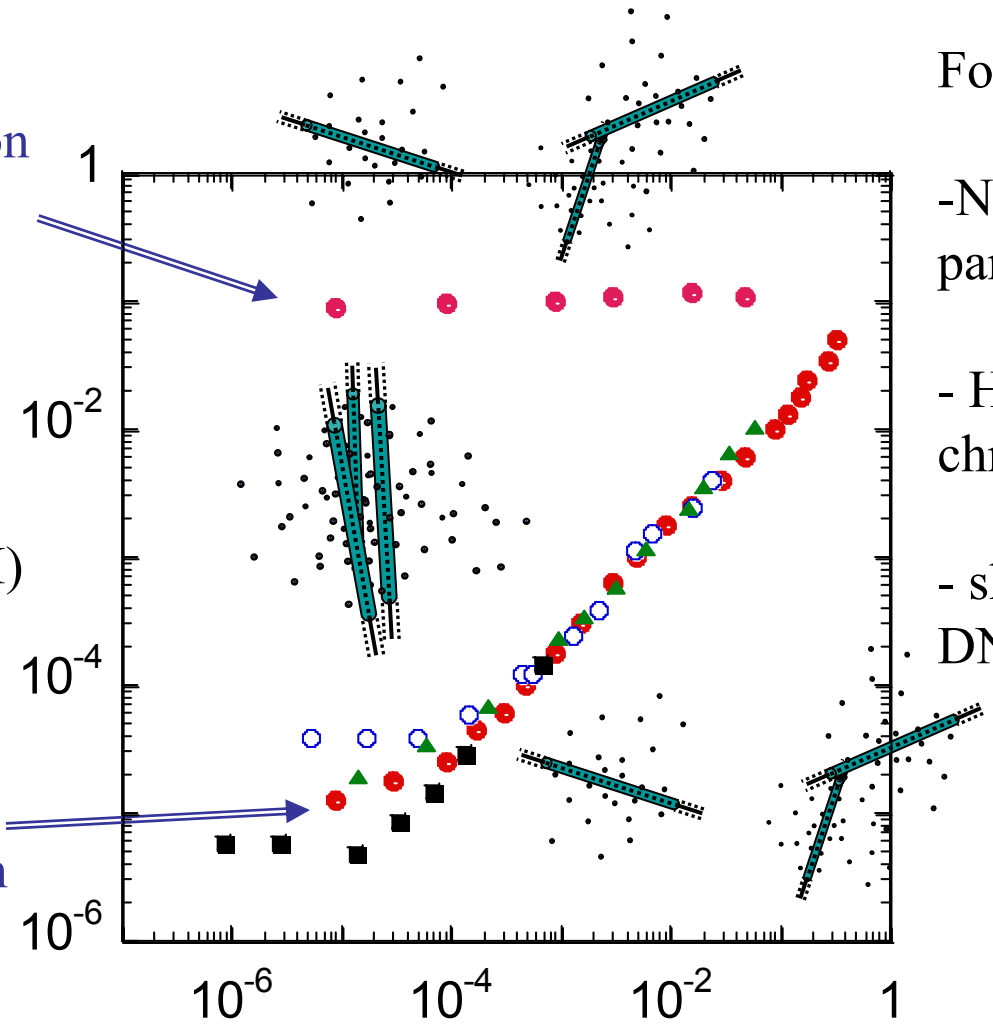
-Nucleosome core particles

- H1-depleted chromatine fragments

- short single-stranded DNA

DNA redissolution
In excess of spermine

DNA precipitation



(Raspaud et al., 1998-99)
(Delsanti et al 94)

Commercial polyelelctrolytes

- Strong Polyelectrolytes:
- Polystyrene sulphonate: if less than 30% of monomers have a sulfur not water soluble, but more than 30% is water soluble.
- Linear charge density : 1 e / 2.5 Å ~ 4 e / nm. if 100% monomers with sulfurs.

- Weak Polyelelctrolytes: the charge density depends strongly on pH of solution, such as polyacrylic acids.

1_ Brief Introduction on the Ionic dissociation and hydration

1_a) simple salt

Coulomb's law (1780)

Attraction force between two elemantary charges e separated by a distance d

$$F = \left(\frac{e^2}{4\pi\varepsilon_0}\right)\times\frac{1}{d^2}$$

and its associated energy :

$$W = \left(\frac{e^2}{4\pi\varepsilon_0}\right)\times\frac{1}{d}$$

For a typical ionic crystal,

$$W \approx 10^{-18} Joules$$

Thermal Energy,

$kT \approx 10^{-21}$ *Joules* (0.6 kcal/mol)

Roughly $W \approx kT$

In water, $\varepsilon = 80$

(In ionic solids is ~ 6 and in air ~ 1)

The salt dissociates in water

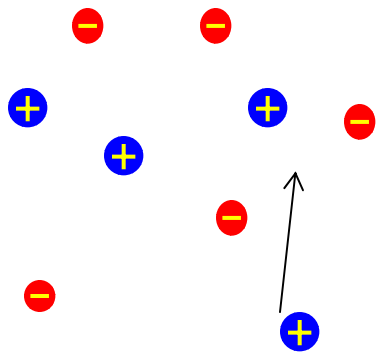
In general, one defines a characteristic length of the ions dissociation/association :

The Bjerrum length l_B

$$kT = \frac{e^2}{4\pi\varepsilon\varepsilon_0 l_B}$$

In water,

$l_B = 7.14$ Å



if $d < l_B$ then
ion pairing occurs

For a multivalent salt

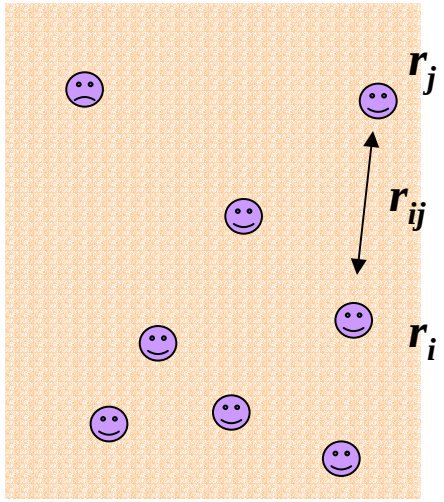
$z_i : z_j$

$|z_i z_j| \; l_B \uparrow$

2_ Simple salts : the Debye-Hückel model (1923)

→ Solution of strong electrolytes (McQuarrie, 1976) :

Positive and negative charges in a continuum medium of dielectric constant $\varepsilon\varepsilon_0$



The Coulombic potential acting upon the ith ion located at r_i

$$\psi_i(r_i) = \sum_{j \neq i} q_j / 4\pi\varepsilon\varepsilon_0 r_{ij}$$

or at the arbritary point r :

$$\psi(r) = \int \rho(r')dr' / 4\pi\varepsilon\varepsilon_0 |r - r'|$$

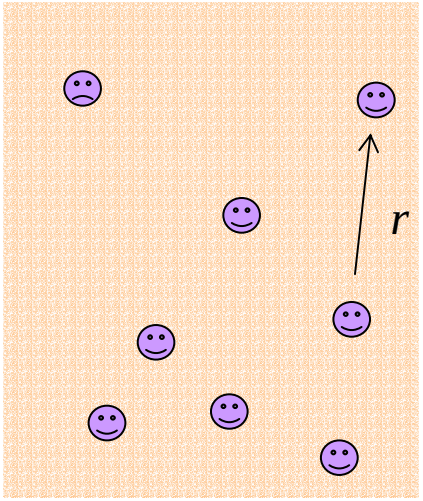
with $\rho(r)$ the total charge density at r

How does the charges distribution $\rho(r)$ change with respect to the distance r from a central ion ?

2_a) spatial distribution

- Ionic size : s
- cations and anions charges : z_+e and z_-e
- concentration of the bulk solution : C_+^0, C_-^0

$$z_+ e C_+^0 + z_- e C_-^0 = 0 \qquad \text{(Electroneutrality)}$$



$\rho(r)$ is related to the electrostatic potential $\psi(r)$ by the Poisson's law and by the Boltzmann's distribution :

$$\Delta\psi(r) = -\rho(r)/\varepsilon\varepsilon_0$$

$$\rho(r) = z_+ e C_+^0 e^{-z_+ e\psi(r)/kT} + z_- e C_-^0 e^{-z_- e\psi(r)/kT}$$

with $F_i = z_i e \psi(r)$ the energy of the charge $z_i e$ located at the distance r from the central ion or equivalently the electrostatic work required to bring the charge from the reference state to the distance r.

Solving the Poisson-Boltzmann equation :

$z_i e\psi(r) << kT$ $\qquad$ $(kT/e \approx 25\text{mV})$

Expanding the exponential, one gets
the linearized Poisson-Boltzmann or the Debye-Hückel equation :

$\Delta\phi(r) = \kappa^2\phi(r)$ $\qquad$ with $\qquad$ $\phi(r) = e\psi(r)/kT$

$\kappa^2 = 4\pi\, l_B(z_-^2 C_-^0 + z_+^2 C_+^0)$

$1/\kappa$: the Debye screening length

Boundary conditions :

$r \to \infty,\ \phi \to 0$

$r = s = a$

, $E = -d\phi/dr = -z_0\, l_B/s^2$

(from the Gauss theorem with
z_0 the valence of the central ion)

$\phi(r) = A\, e^{-\kappa r}/r$

a screened coulombic potential

with $A = (z_0\, l_B\, e^{\kappa s})/(1+\kappa s)$

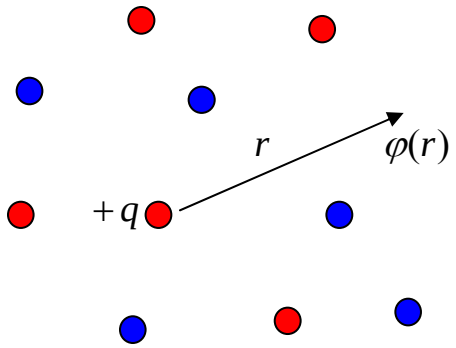
Point ions fluid

Linearized Poisson-Boltzmann equation

$$\Delta\varphi = \kappa^2\varphi$$

$$\kappa^2 = 4\pi q^2 l_B (\rho_+ + \rho_-)$$ ionic strength

$$l_B = \frac{e^2}{\varepsilon k_B T}$$ Bjerrum length



$$\frac{F_{el}}{k_B T V} = -\frac{\kappa^3}{12\pi}$$

2_b) thermodynamic consequences

Knowing the ion spatial distribution and the electrostatic potential and charging the ions, one can calculate the electrostatic free energies of the system :

Helmholtz electrostatic free energy

$$F^{elec} / V\, kT = -(\kappa^3/12\pi)\ \tau(\kappa a)$$

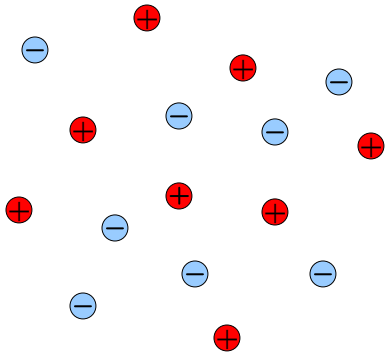
$F^{elec} < 0$ due to correlations

Predict then the thermodynamic coefficients

$$F = F_{ref} + F_{el}$$

→ Phase transition!!!

Electrolyte Primitive Model



N - total number of hard spheres

V - volume of the system

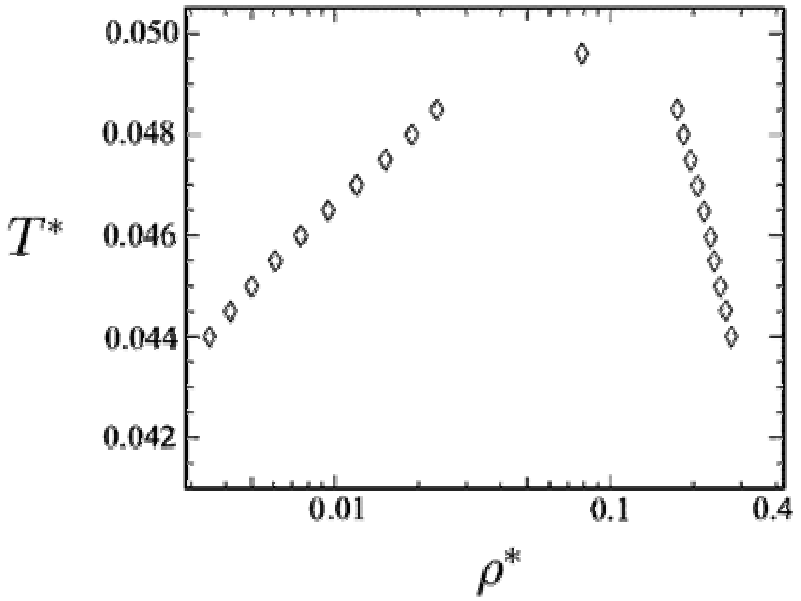
a - diameter of the sphere

q - charge of the sphere

$$T^* = \frac{k_B T a}{q^2}$$

$$\rho^* = \frac{\mathrm{N} a^3}{V}$$

Monte Carlo Simulations

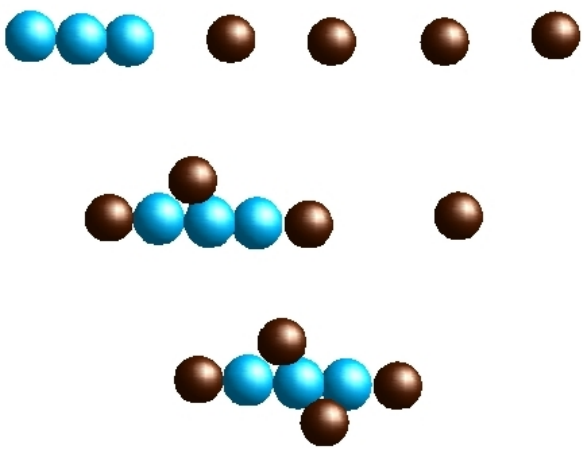


Z. Panagiotopoulos, M. E. Fisher, Phys. Rev. Lett. 88, 045701 (2002)

T* low (l_B/a high) → D-H breaks down.
Ion paring and cooperative interactions.

In monovalent salts chains + and – pairs are formed

In multivalent salt clusters of 3+ and -



D-H also breaks down if φ increases $d \sim (1/e\sigma)^{1/D} \; l_B$

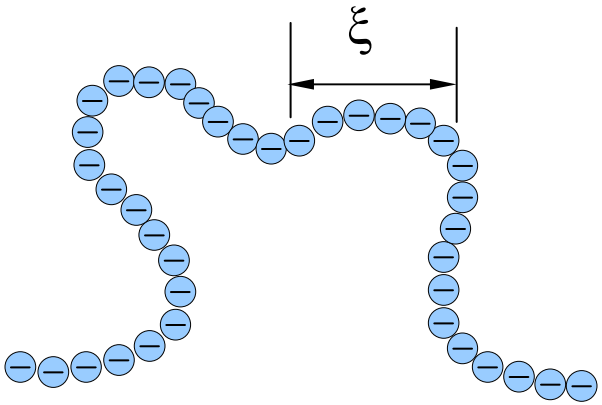
Low salt polyelectrolyte solutions

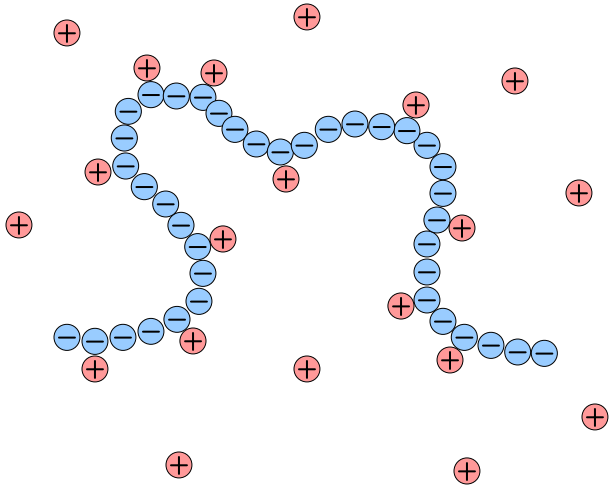
In polyelectrolytes the linearized theories are not valid because the chains are perturbed by electrostatic interaction

$$\xi \sim 1/\kappa \approx (\rho^*)^{-1/2}$$

They are stretched due to the repulsions

deGennes et al, 70's; Kremer + Stevens 1995; A. V. Dobrynin, M. Rubinstein, 1999, 2003;

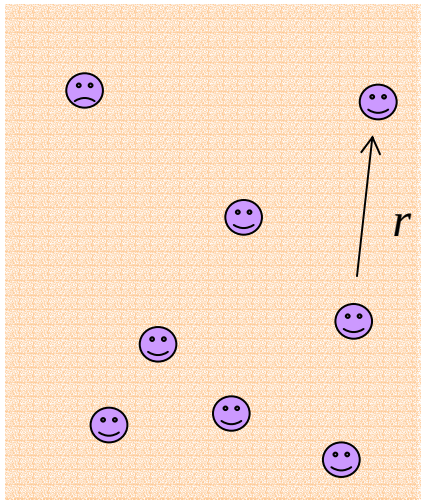




BUT, there is ion association along the chains: Shapes depends on how many ions are around the chains (Gonzalez-Mozuelos et al 95; Liu + Muthukumar, 02)

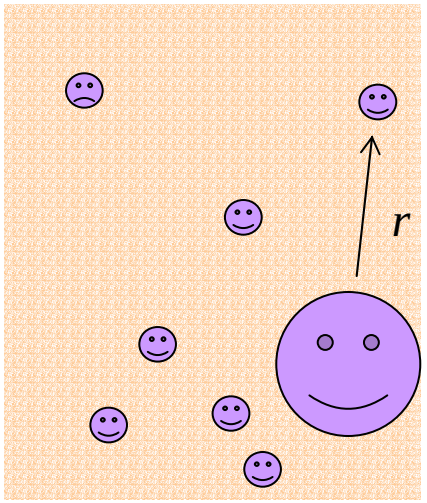
3_ Polyelectrolyte and the counterions distribution

Simple salt



Apply similar calculation ?

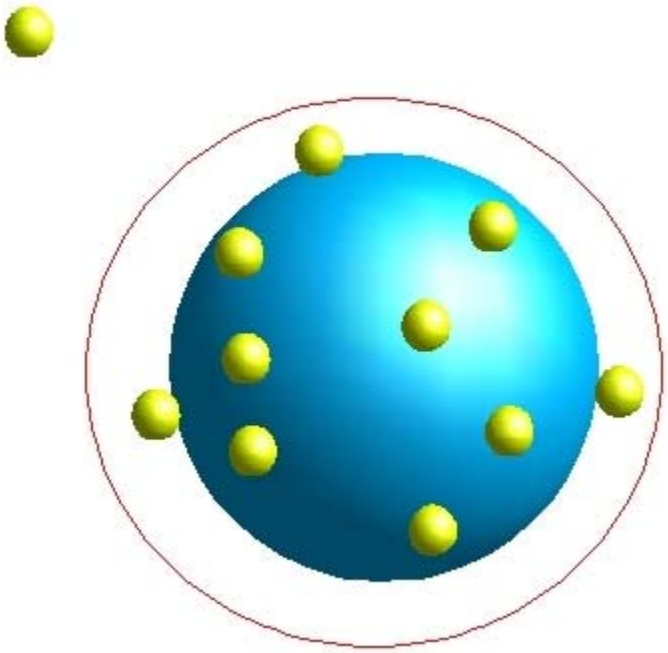
A polyelectrolyte into an ionic solution



-Yes, we can start from the Poisson-Boltzmann equation

- No, the approximation $z_i e \psi(r) << kT$ is no more valid

In colloids the"condensed"ions renormalized the charge like a double layer in charged surfaces.
Away from the macroion the interaction with other charges reflects only the effective charge of the macroion, and follows mean field values (Debye-Huckel, DVLO).



Warning. Poisson Boltzmann ignores short range correlations important in the dense ionic region or double layer.

3_a) the Poisson-Boltzmann equation

The central ion becomes the polyelectrolyte :

$\psi(r)$ its electrostatic potential at a distance r

$\rho(r)$ the charge density of the surrounded ions

$\Delta\psi(r) \sim \rho(r)$

$\rho(r) \sim e^{-ze\psi(r)/kT}$

$\Delta\psi(r) \sim e^{-ze\psi(r)/kT}$

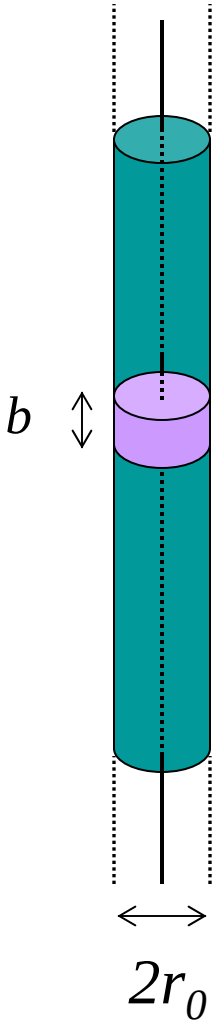
In cylindrical geometry,

$\Delta\psi(r) = (1/r)\,\partial(r\ \partial\psi/\partial r)/\partial r$

a non-linear differential equation

Number of charges per Bjerrum length l_B :

$\xi_M = l_B / b$ the Manning parameter

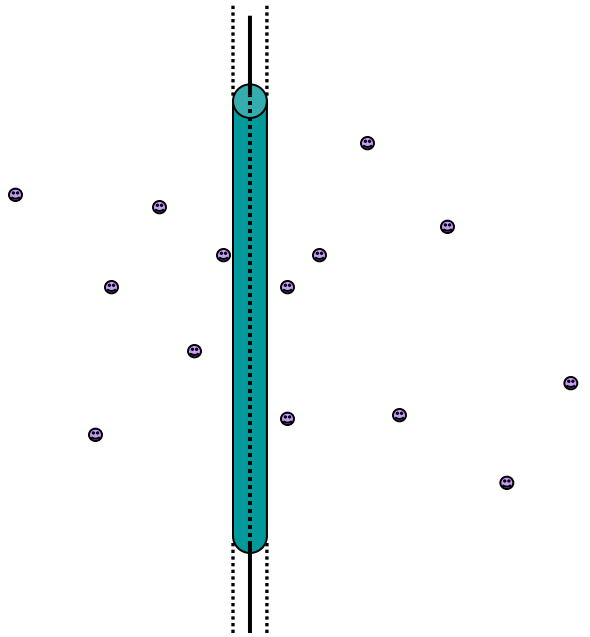


For B – DNA, b = 1.7 Å : ξ_M = 4.2

Without added salt :
Only the counterions are present

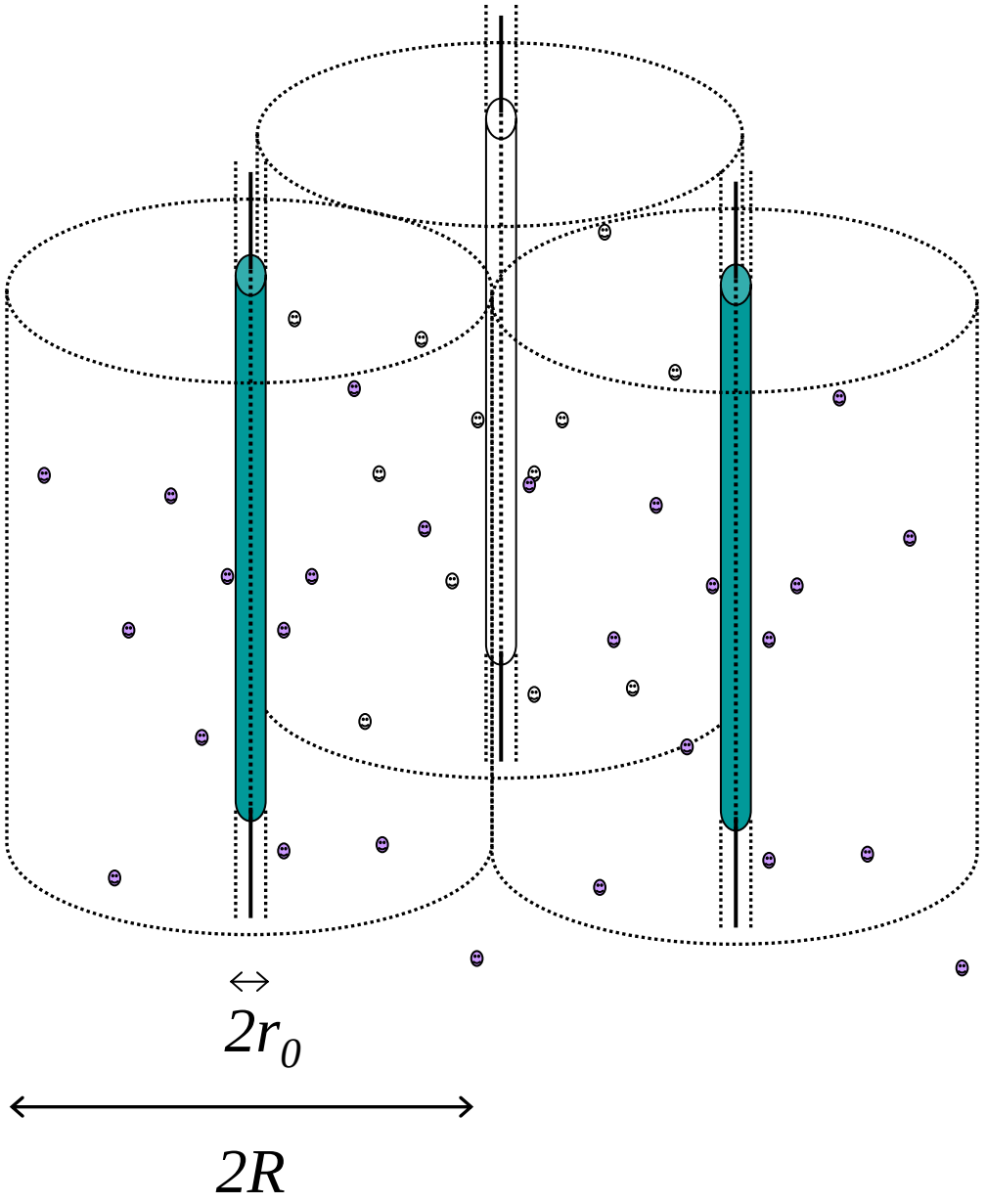
$z_c\, e\, C_c^{total} + z_m\, e\, C_m^{total} = 0$

Counterions	Monomers
$z_c = +1$	$z_m = -1$
monovalent	a nucleotide
(Na^+, K^+,…)	($Phosphate^-$)



Counterions distribution ?

The Cell model : each chain enclosed in a coaxial cylindric cell



- <u>ionic distribution :</u>

the whole solution

=

a single cylindric cell

- <u>concentration effect :</u>

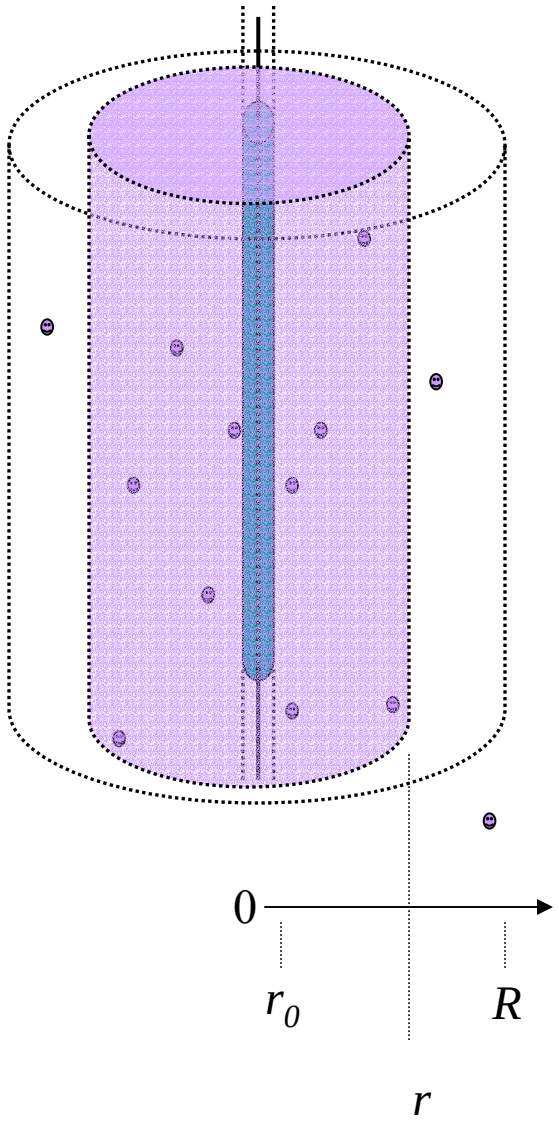
DNA dilution

=

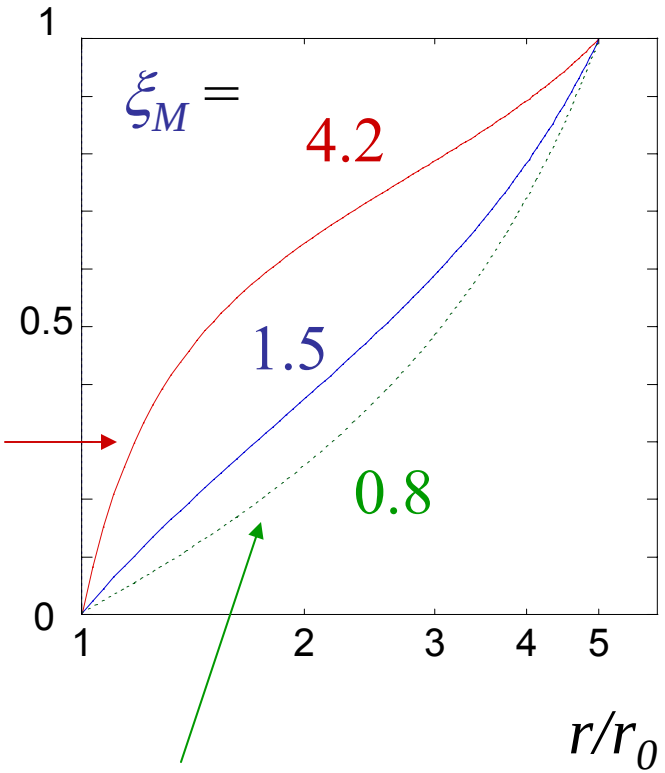
R increase

$C_m^{total} = C_c^{total} = 1/\pi bR^2$

Fraction of counterions within a cylinder of radius r



$R = 50$ Å



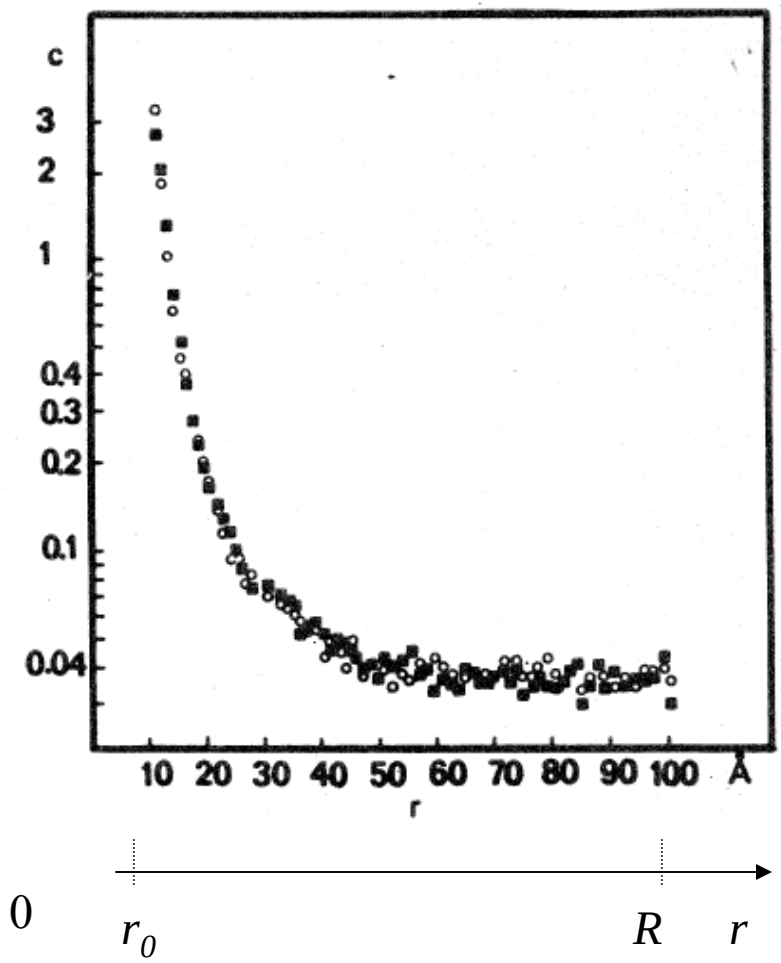
Counterions accumulation

Monotonic increase for $\xi_M < 1$

Monte-Carlo simulation (Le Bret & Zimm, 1984) :

C (r) (M)

local concentration
of countreions



- regular spacing of the phosphate charge on a cylinder

-charges set along the double-helix

Experimentally :

X-Ray scattering (Chang et al., 1990)

Heavy counterions (Thallium$^+$)

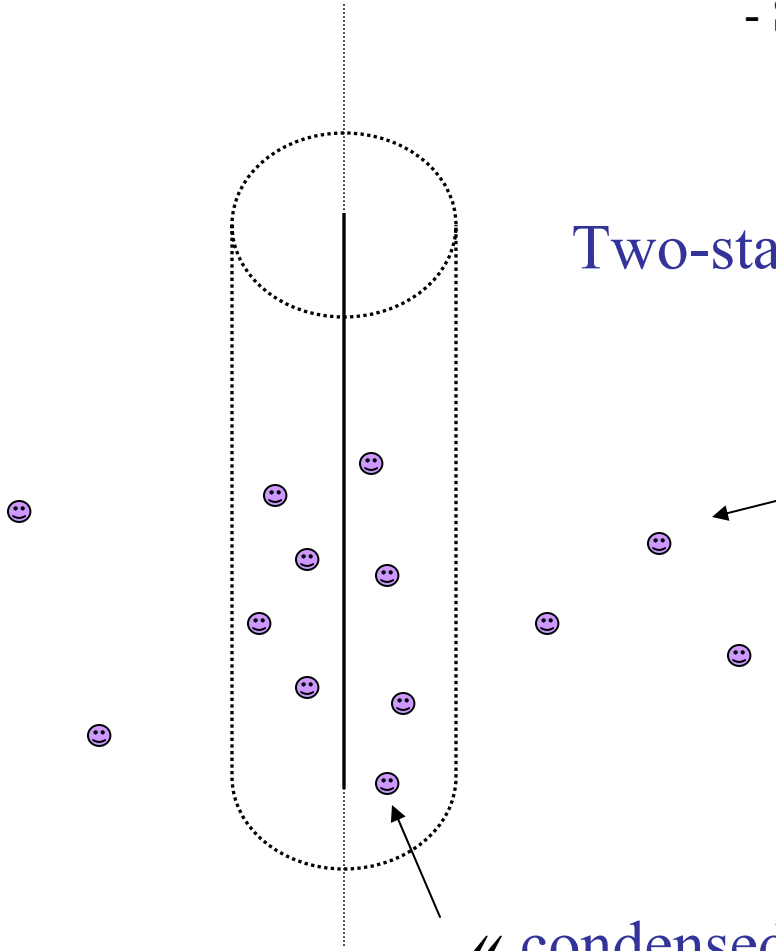
Neutron scattering (van der Maarel et al., 1992)

Contrast matching

3_b) the Manning model and the limiting laws

(Manning, 1969; 1978)

-An infinite line charge with a charge density ξ_M
- Simple salt is present in excess over polyelectrolyte

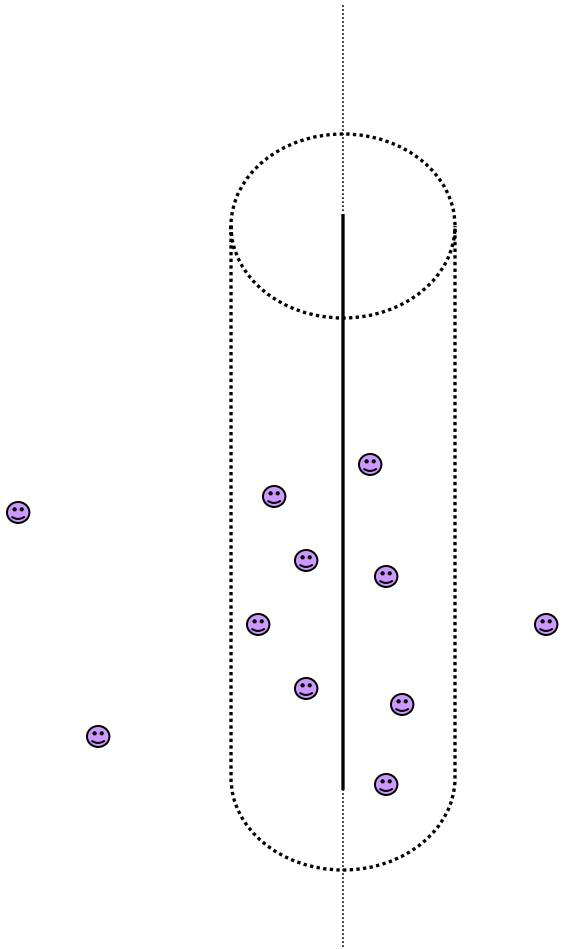


Two-states model for $\xi_M > 1$:

« free » ions treated in the Debye-Hückel approximation

« condensed » ions reducing the monomer charge to an effective charge $q_{eff.}$

Calculation of the effective charge $q_{eff.}$:



-The condensed ions decrease the repulsions between the chain charges :

$$f_{el.} = (q_{eff.}^{\ 2} / 4\pi\varepsilon\varepsilon_0) \Sigma\, e^{-\kappa r_{ij}} / r_{ij}$$

By summing over all pairs of effective charges of the chain, the electrostatic contribution to the free energy per monomer becomes for $\kappa b << 1$:

$$f_{el.} = - (q_{eff.}^{\ 2} / 4\pi\varepsilon\varepsilon_0 b) \ln \kappa b$$

Set θ the number of condensed ions per monomer, the effective charge per monomer becomes:

$$q_{eff.} = z_m e + z_c e\, \theta = z_m e\, [1 + (z_c/z_m)\theta]$$

with $z_c z_m < 0$

$$\longrightarrow \quad f_{el.} / kT = - z_m^{\ 2} [1 + (z_c/z_m)\theta]^2\, \xi_M \ln \kappa b$$

Minimization of the free energy $f_{el.} + f_{mix.}$ with respect to θ :

$$1 - \ln(V_p C_s/10^3) - z_c z_m [1+ (z_c/z_m)\theta] \xi_M \ln (\kappa b)^2 = 0$$

Focus on the salt concentration C_s dependence with $\kappa^2 \sim C_s$

$$- \ln C_s \sim z_c z_m [1+ (z_c/z_m)\theta] \xi_M \ln C_s$$

The only solution to cancel the C_s dependence (in particular when $C_s \longrightarrow 0$) :

$$\boxed{-1 = z_c z_m [1+ (z_c/z_m)\theta] \xi_M}$$

For $| z_m | = | z_c | = 1$ $\longrightarrow$ $\boxed{\theta = 1 - 1/\xi_M}$

4_ Thermodynamic coefficients and the « free » counterions

« free » in the Manning sense :

For $\xi_M < 1$, all the counterions are « free »
For $\xi_M > 1$, only $1/\xi_M$ counterions are « free »

But they are submitted to the electrostatic potential created by the DNA chains;
Similar to the ionic atmosphere of the Debye-Hückel interations for simple salts

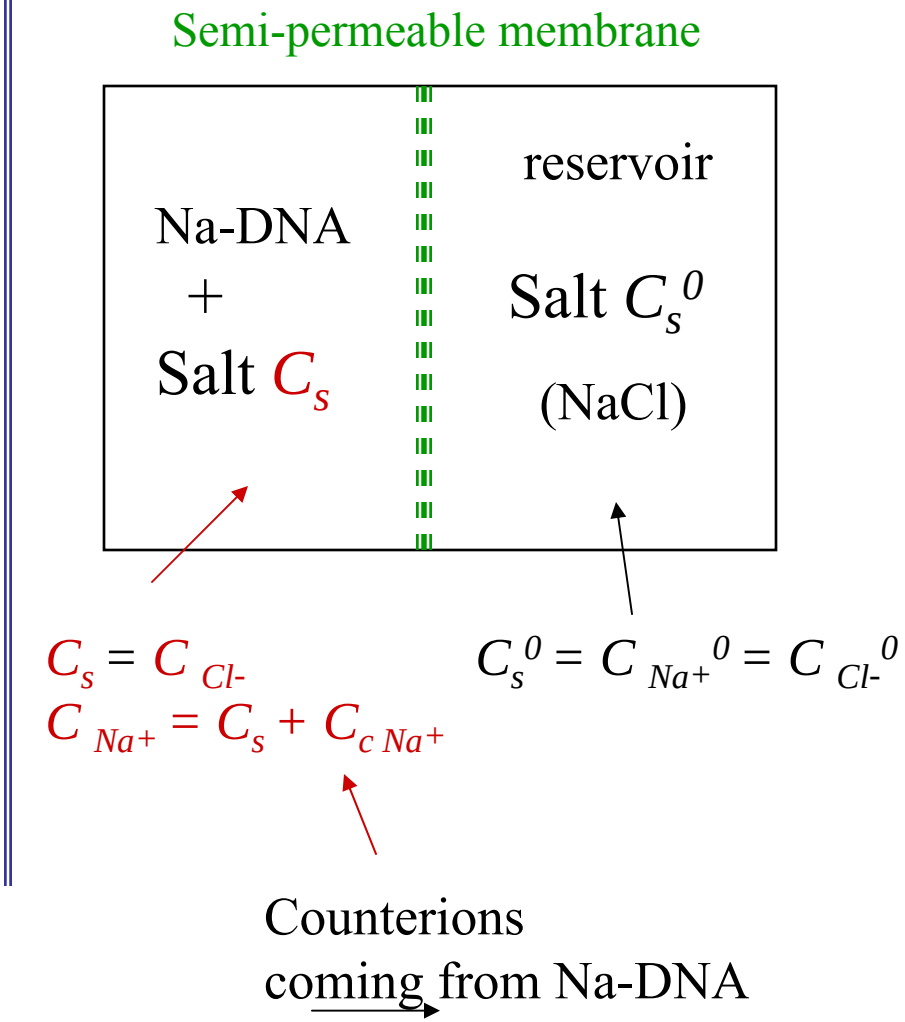
≠ thermodynamically « free » : where ions are only submitted to the thermal agitation

ONLY is Salt is MONOVALENT

The Donnan effect :

At the thermodynamical equilibrium,
the ions chemical potentials
from both sides are equal :

$$\mu(C_{c\,Na^+}, C_s) = \mu(C_s^0)$$

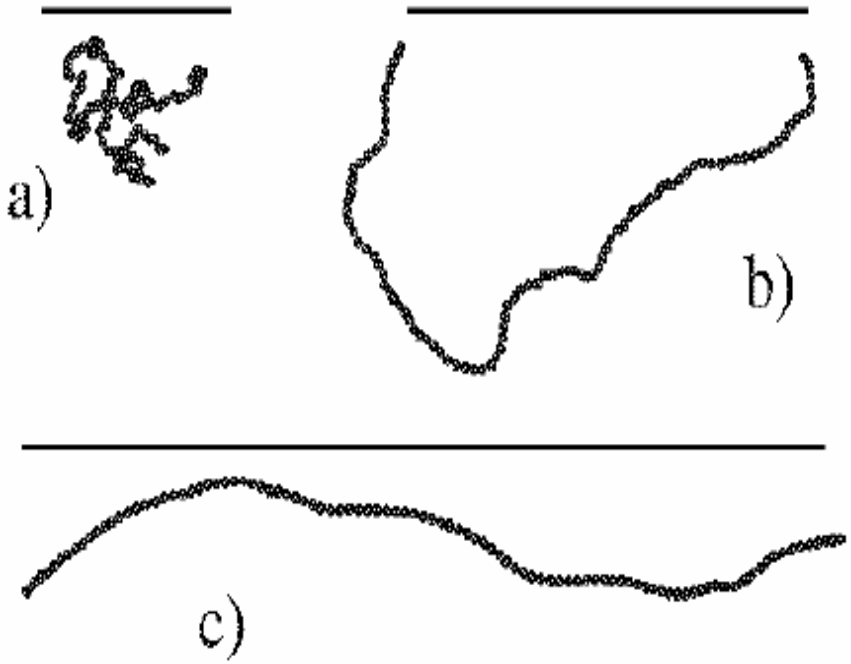


lim DNA~0, $2\Gamma = 1/2\xi_M$

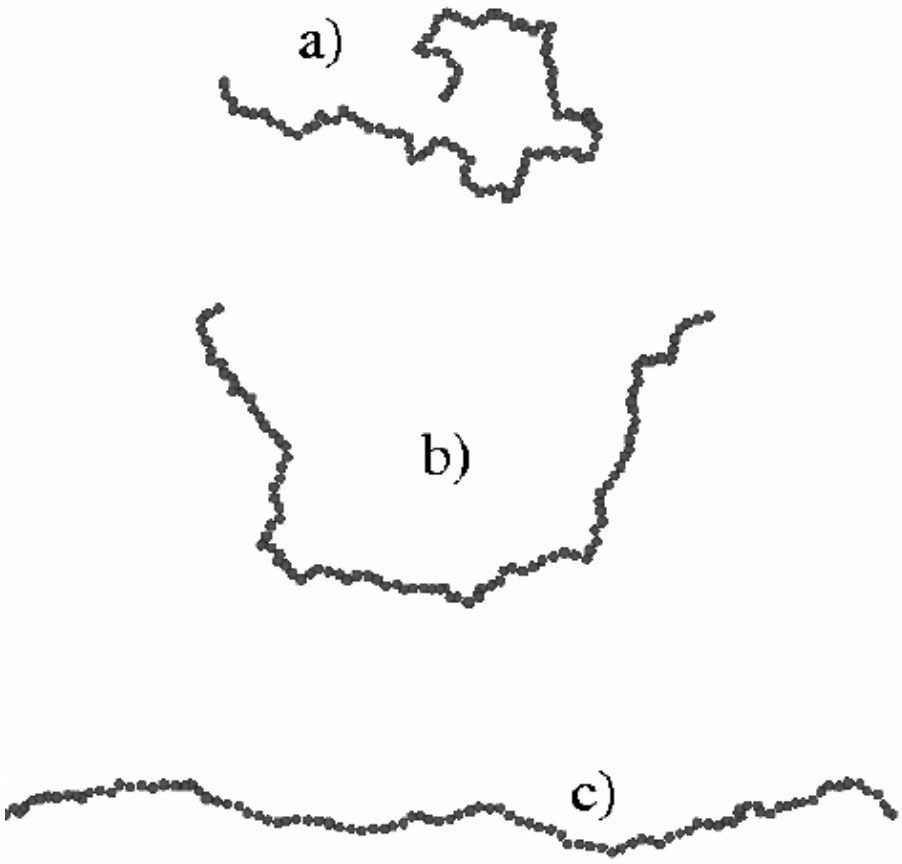
INFINITE RODS ONLY

Information on the fraction of thermodynamically « free » counterions

Chain models: Neutral flexible a) to rigid chains c)



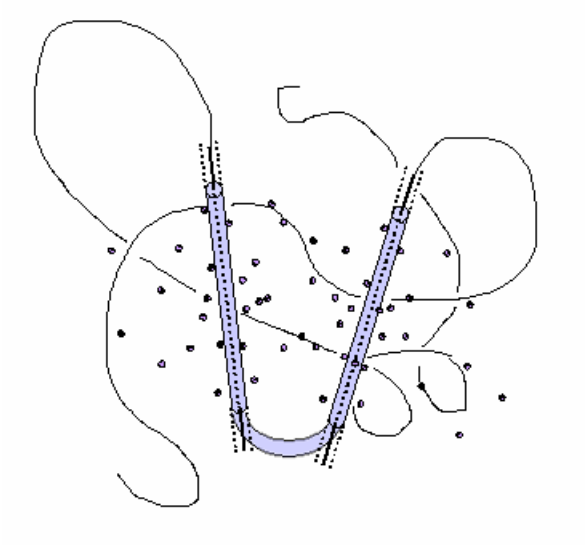
Charged flexible chains high a) to low c) salt concentration (screened Coulomb interactions)



Netz + Andelman 2002

5_ Conformation and Interaction

Intramolecular interactions :



The persistence length L_p : the bending flexibility

Bending makes the phosphate charges lie closer from one another : electrostatic contribution to L_p

$$L_p = l_0 + l_{ele}$$

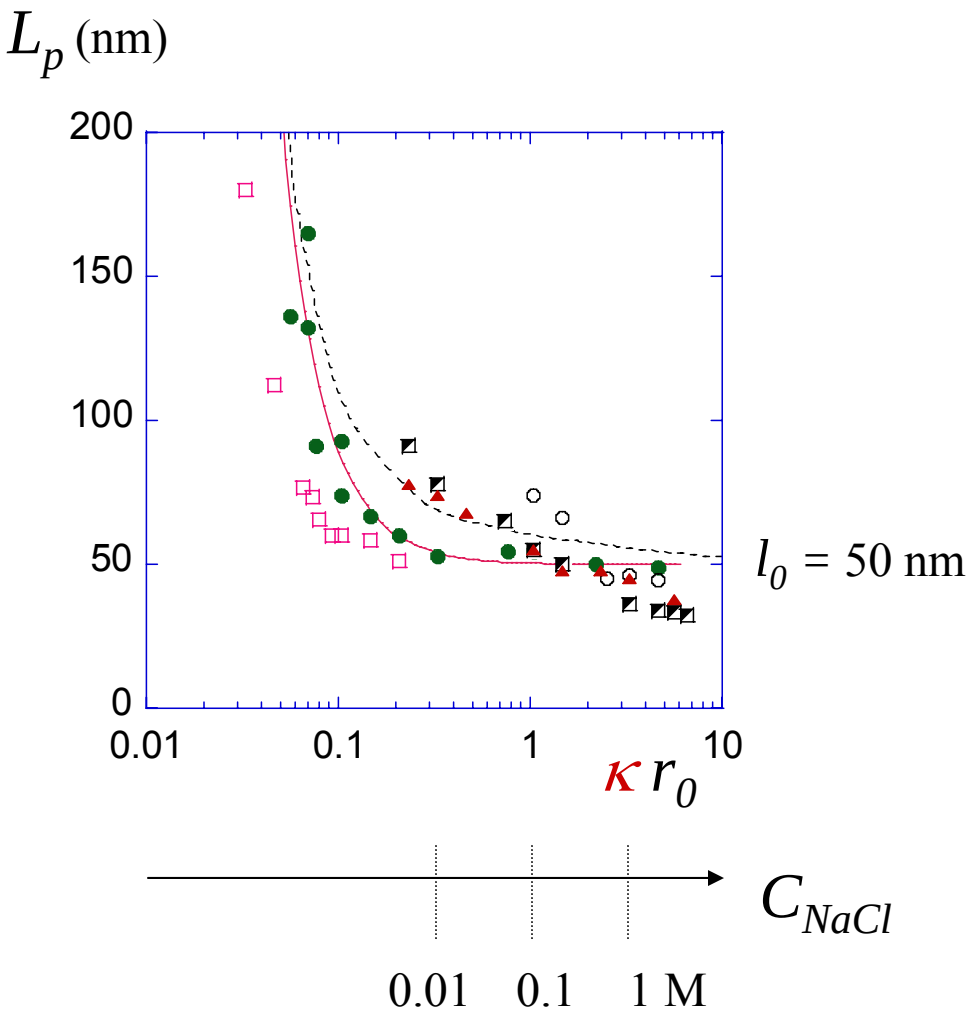
- Using the Debye-Hückel approximation and the limiting law ($\kappa L_p >> 1$),

$$l_{ele} = l_{OSF} = l_B / 4\,(\kappa b)^2 \quad \text{with} \quad b \longrightarrow l_B$$

(Odijk, 1977; Skolnick & Fixman, 1977)

- Using the P-B equation, $l_{ele} = l_{OSF}\, f(\kappa r_0)$

(Fixman, 1982; Le Bret, 1982)

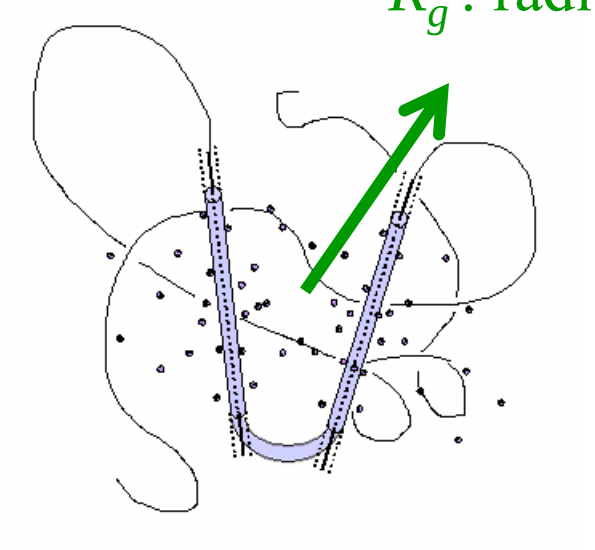


$L_p = l_0 + l_{OSF}$

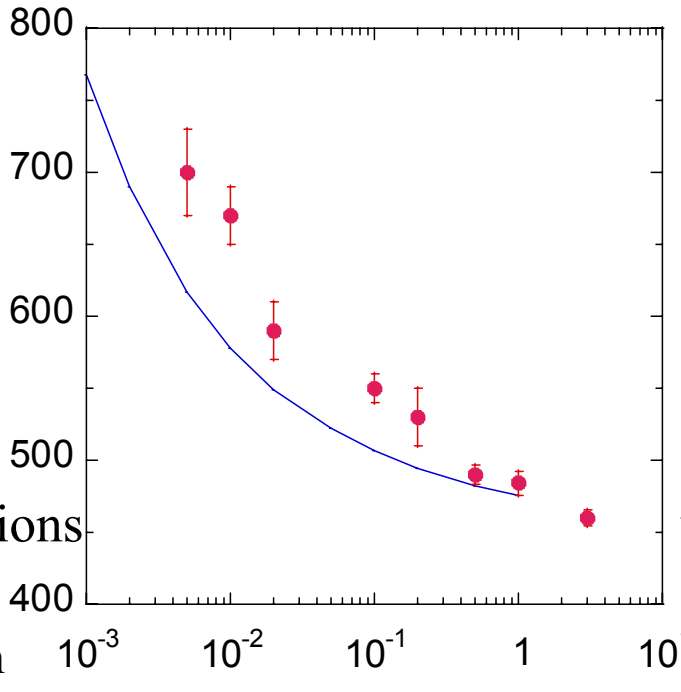
$L_p = l_0 + l_{OSF}\, f(\kappa r_0)$

- 6360 bp (Maret & Weill, 1983)
- T2 DNA (Harrington, 1978)
- 587 bp (Hagerman, 1981)
- T7 DNA (Sobel & Harpst, 1991)
- linear Col E1 plasmid (Borochov et al., 1981)

R_g : radius of gyration



R_g (nm) of T7 DNA (40 10^3 bp)



Experimental data
(Sobel & Harpst, 1991)

← Random coil

Including :

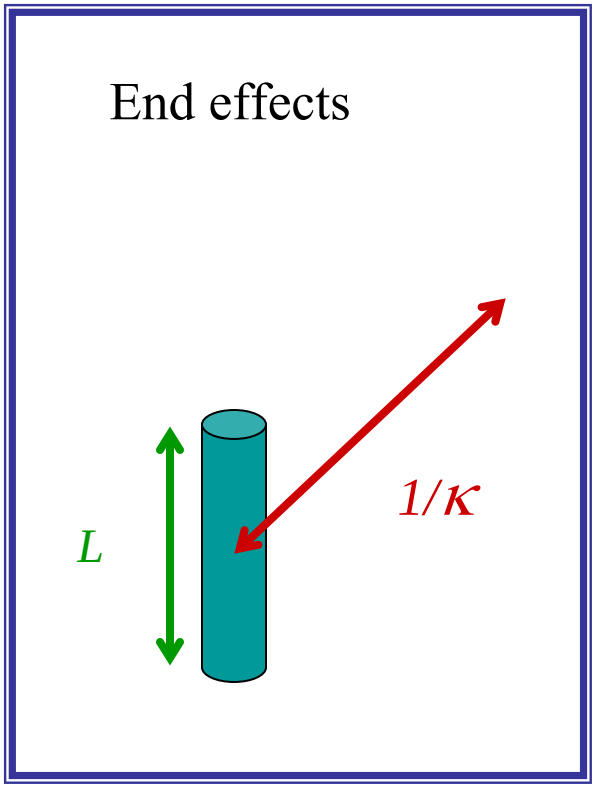
- excluded volume
  with d_{eff} due to repulsions
- persistence length
  with $l_{OSF}\ f(\kappa r_0)$, l_0 = 40 nm

(Stigter, 1985)

the length and flexibility effect

Counterions evaporation

(Alexander et al.,1984)



For charged spheres, ion chemical potential :

far from the spheres, mainly entropic $kT \ln C$
on its surface, mainly enthalpic (electrostatic)

$$-q_{eff} e^2/4\pi\varepsilon\varepsilon_0 a$$

$$q_{eff} \sim - \ln C$$

The effective charge increases with dilution

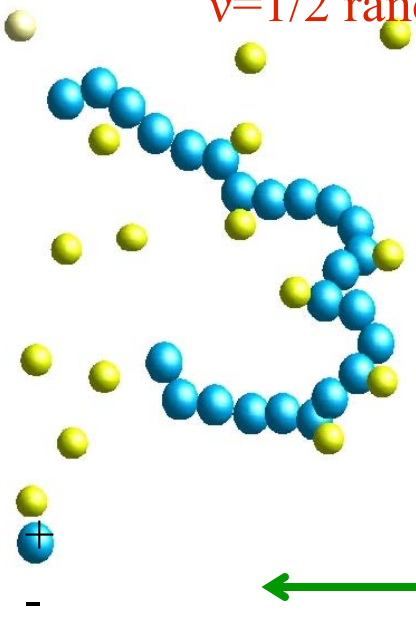
→ Counterions evaporation

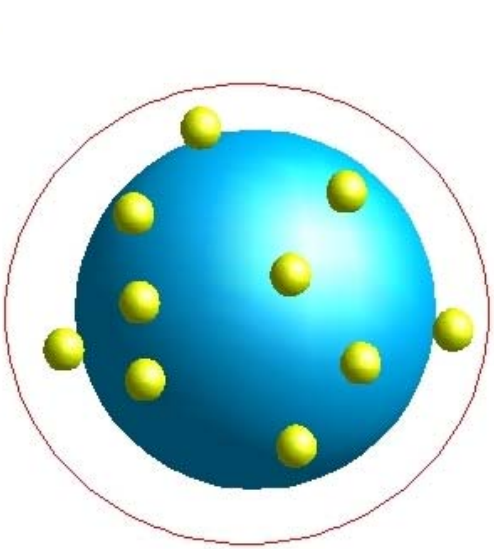
(Gonzalez-Mozuelos & Olvera de la Cruz, 1995)

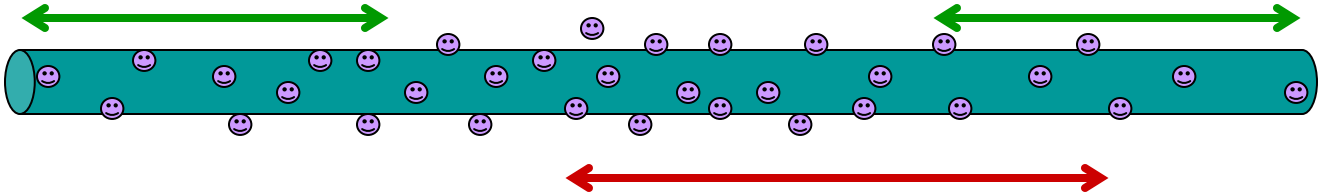
PB similar to equate chemical potential of free and condensed ions to determine the effective charge of the colloid.
This can be done for any input charge distributions. For a fractal chains we input the distribution from the center of mass and allow the ions to "penetrate" the fractal.

$R \sim N^{\nu}$ $\nu = 1$ rod

$\nu = 1/2$ random walk





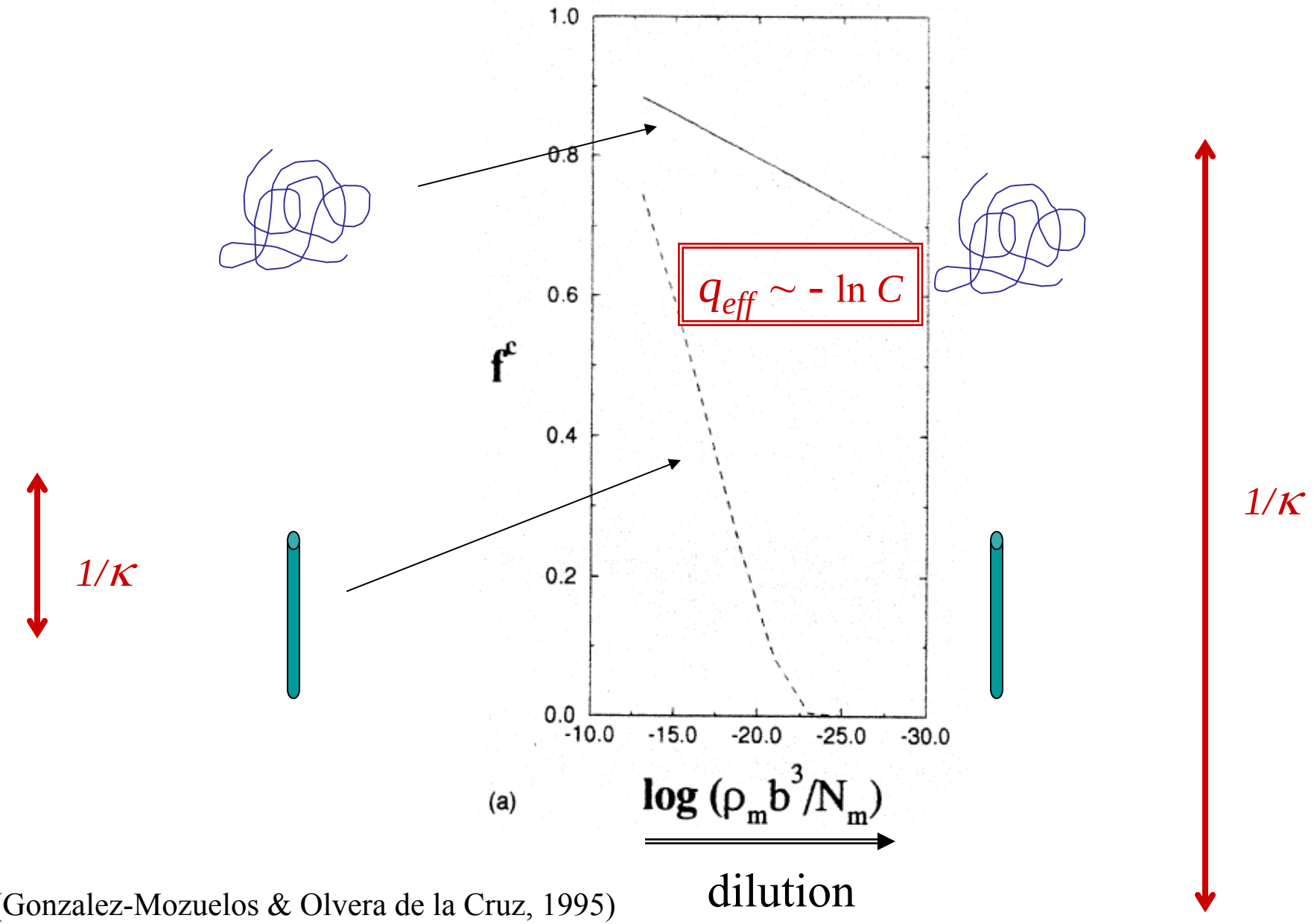


DIFFERENT IN FLEXIBLE

$1/\kappa$

(Olmsted et al., 1989)

fraction of condensed counterions



dilution

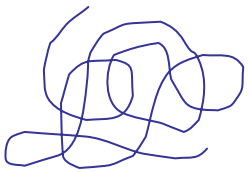
(Gonzalez-Mozuelos & Olvera de la Cruz, 1995)

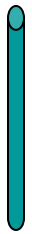
Poison-Boltzmann gives always rod are lowest energy **even though** more ions are condensed in collapsed than in rod.

WITHOUT SHORT RANGE CORRELATIONS

no counterion induced attractions

Z=4.



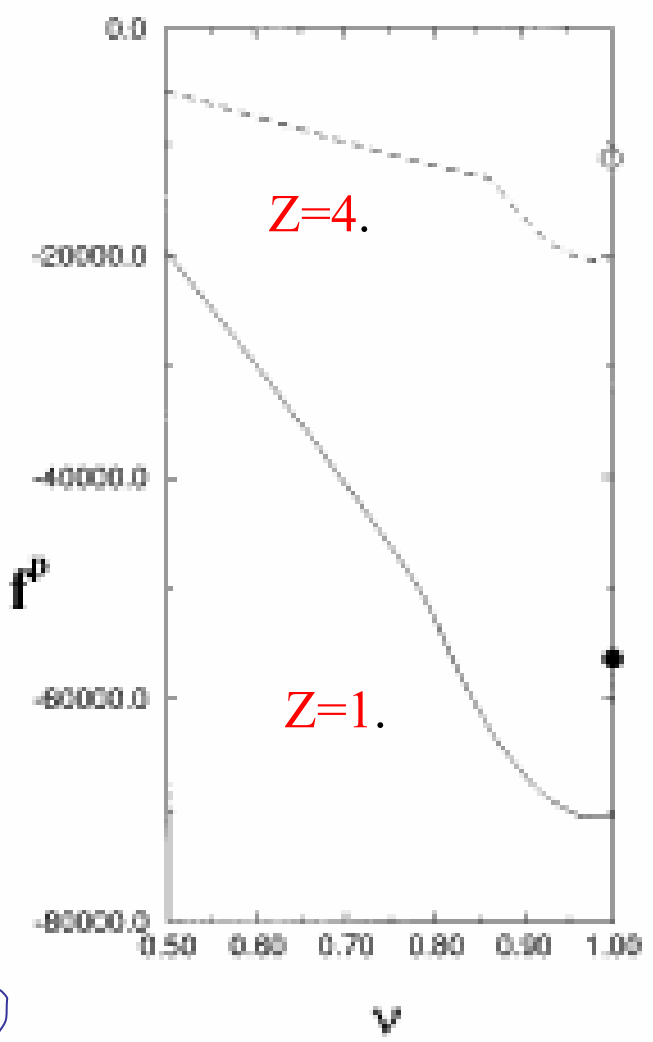


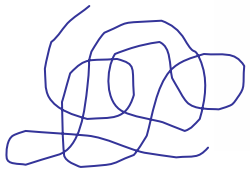
Z=1.

In monovalent and multivalent counterions without correlations in elelctrostatics (~Poison-Boltzmann)

Though more ions are condensed in random walk than in rod the free energy per chain is lowest for rod.

For $l_B/b \sim 1$ to 7 even with some correlations rods have lowest energy in monovalent counterions due to entropy of free counterions.



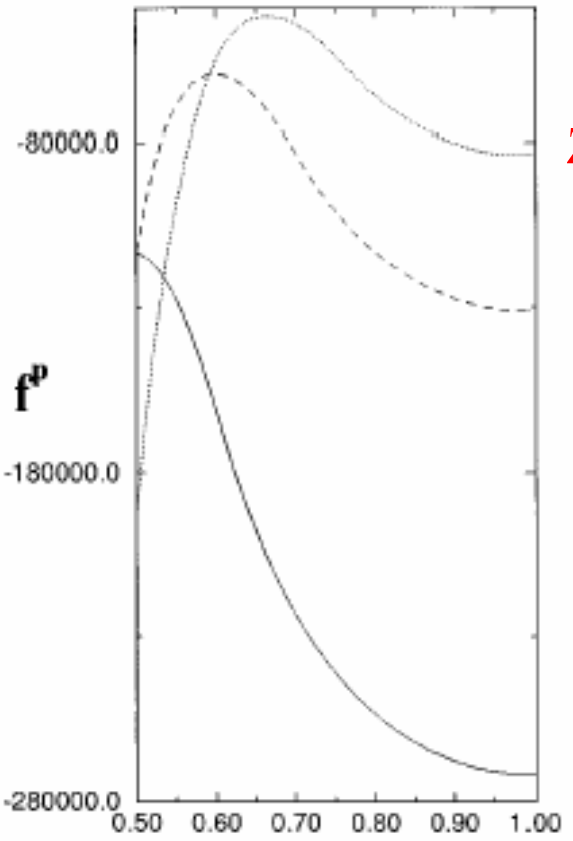


With short range correlations (virial) free energy of compact chains is lower in multivalent ions or monovalent if $l_B/b >> 1$. Since the lowest is for dense sphere→ monomolecular precipitation of chains in multivalent ions or low T*

.

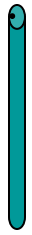
The virial approach is not valid for dense ionic collapse chain ν=1/3; solid state models can describe the energy (Solis et al, 2000, 2001; S. Liu and M. Muthukumar, 2002).

In monovalent ions there is no collapse at T* > 0.1.



Z=3.

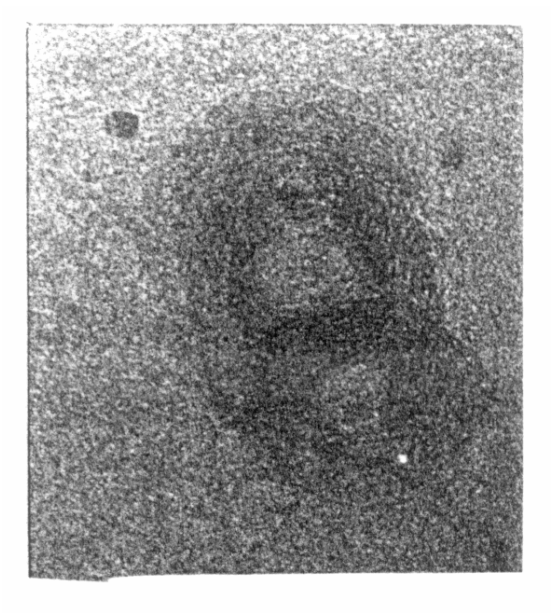
Z=1.



6_ Multivalent Counterions

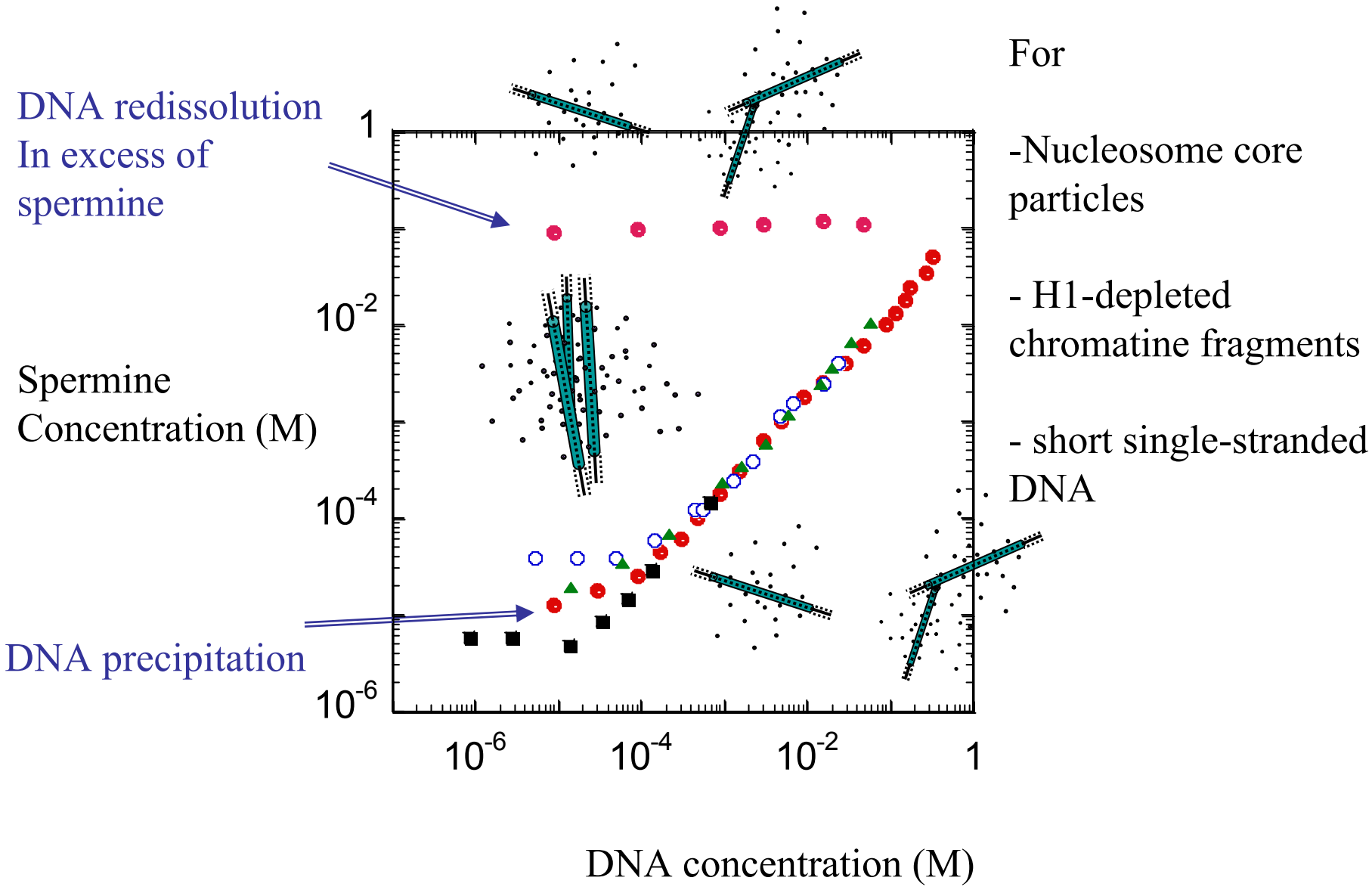
6_a) collapse and phase diagram

For higher DNA concentrations : aggregation



Cryo-electron microscopy

Phase diagram



For

-Nucleosome core particles

- H1-depleted chromatine fragments

- short single-stranded DNA

(Raspaud et al., 1998-99)

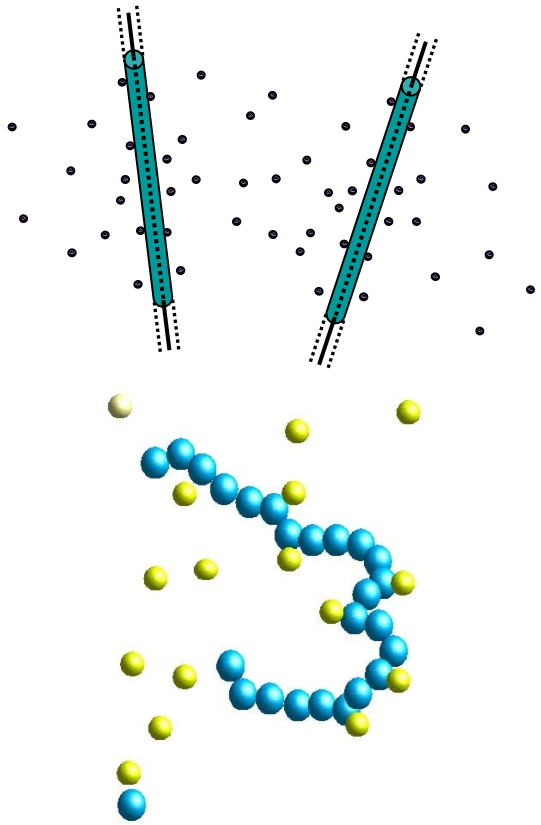
Attractive interaction : in violation of the basic P-B theory

Charge fluctuations (MRPA) NOT enough to predict segregation at room T

- a simple charge (quasi)-neutralization with a non-Coulombic attractive force ?

doesn’t explain the redissolution in excess of multivalent salt

Phase separation of polyelectrolytes with the addition of multivalent ions

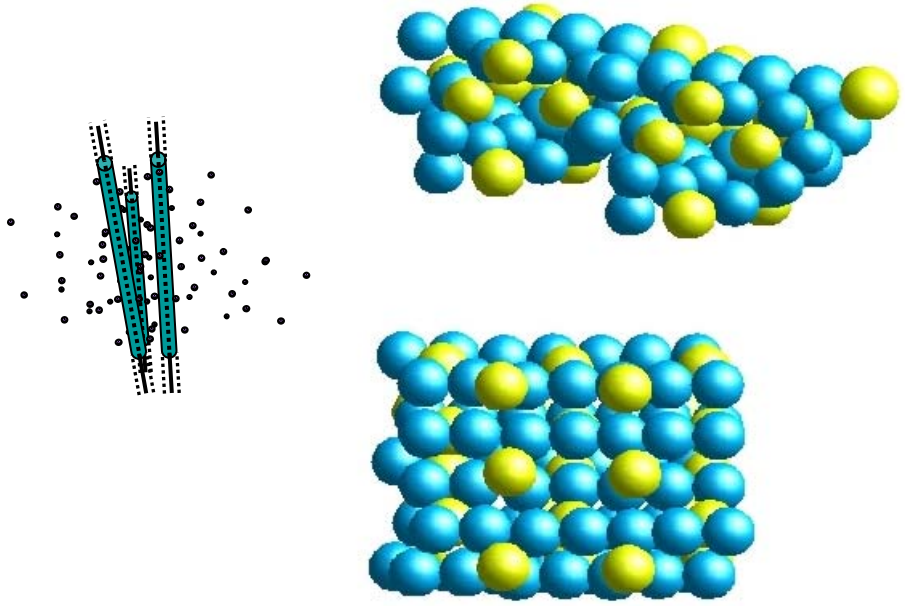


$e^{-\kappa r}$ Repulsions only in dilute with point ions

(Brenner & Parsegian, 1972)

In dense systems

Highly correlated ions leads to attractions (Modified RPA gives no transition in salt solutions)



Ionic glass

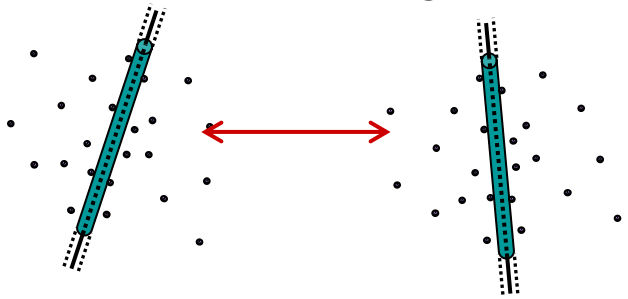
(Solis + M.O. de la Cruz, 2000, 2001)

- a salt bridge model : « ion-bridging » model ?

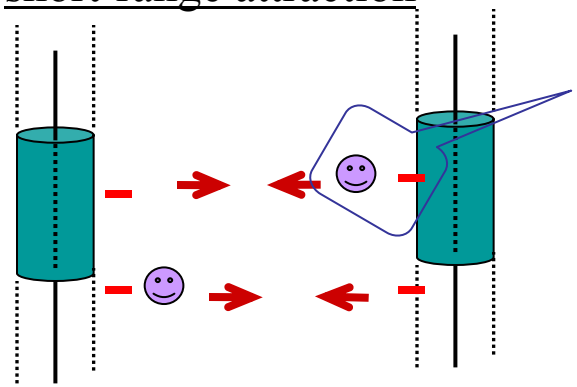
(Olvera de la Cruz et al., 1995)

Intermolecular interactions in dilute solutions with an excluded volume ν :

electrostatic repulsion
between like-charged chains

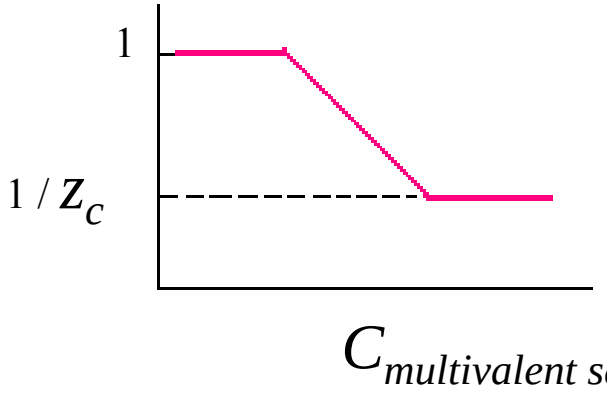


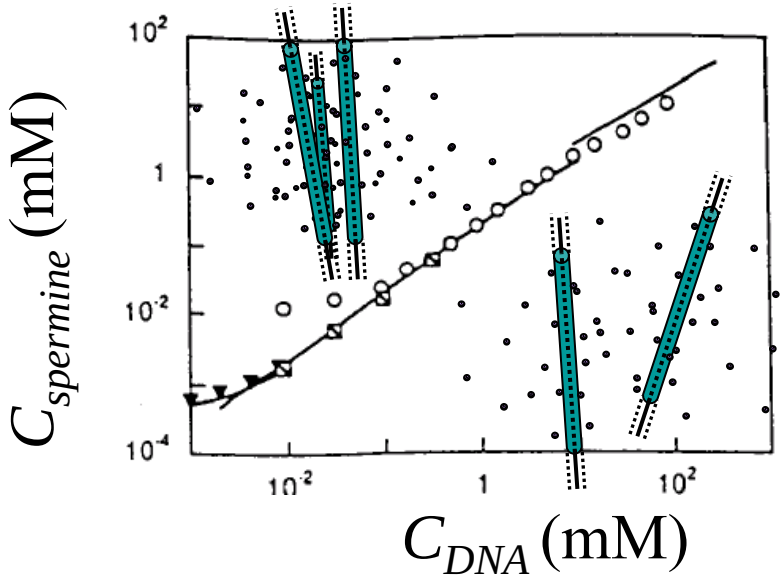
short-range attraction



local
charge
inversion

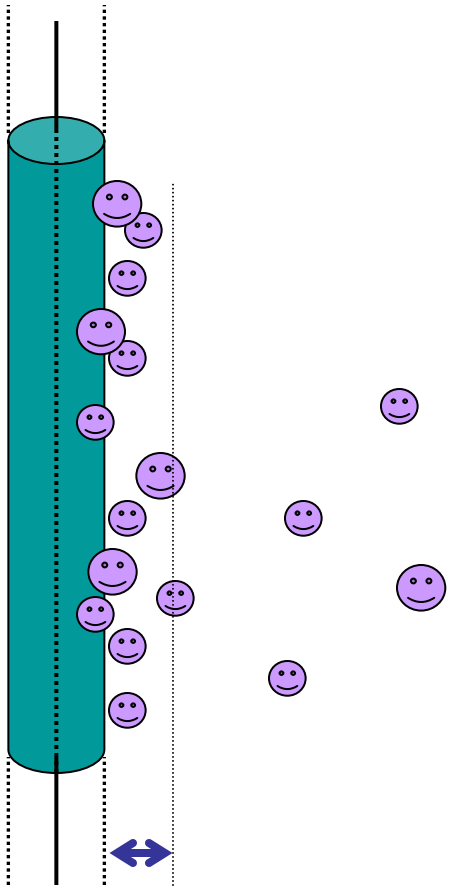
The reduced effective charge : - q_{eff} ξ_M

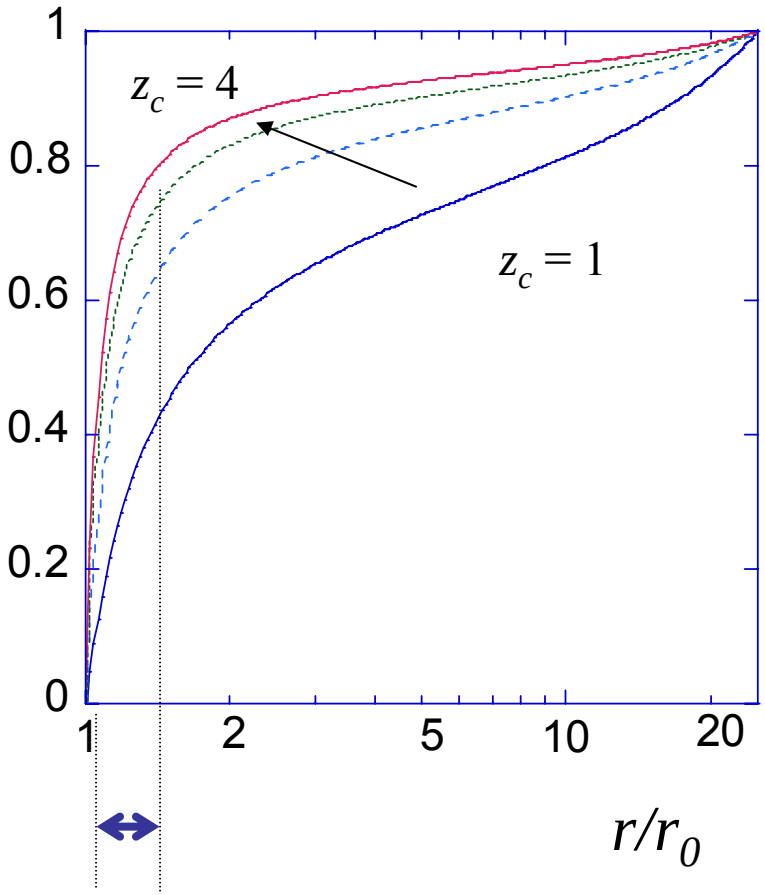




6_b) ionic correlation

Fraction of counterions
within a cylinder of radius *r*
(from the P-B equation)

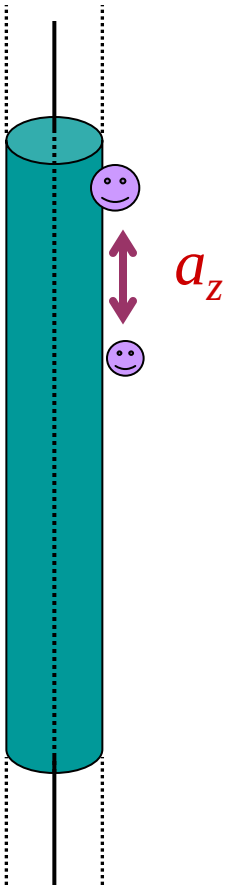




Strong charge accumulation
near the surface (~ one layer)

Few Angstroms

Coulomb energy between neighboring ions



$$W_c = (z_c e)^2 / 4\pi\varepsilon\varepsilon_0 a_z$$

Coupling parameter :

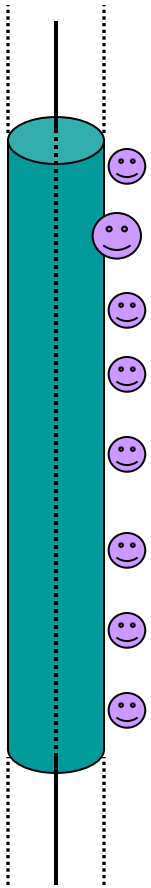
$$\Omega = W_c / kT$$

For multivalent ions, $z_c > 1$ and $\Omega > 1$ strong coupling

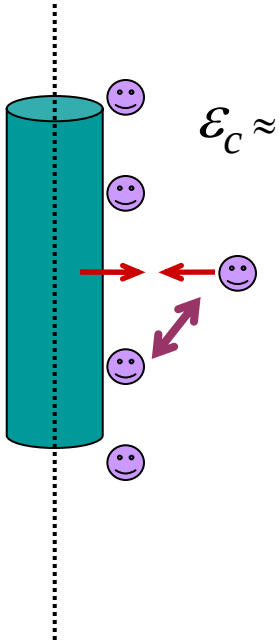
Important longitudinal correlations between ions
« strongly correlated liquid »

(Rouzina & Bloomfield, 1996; Grosberg et al., 2002 and references therein)

Two-dimensional lattice of the multivalent counterions around the surface

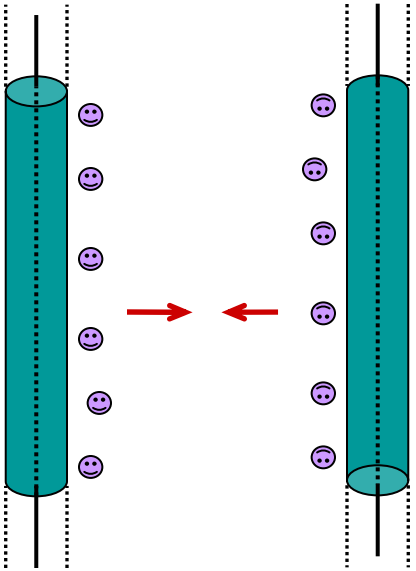


Correlation Energy ε_c per ion < 0 (attractive)



$$\varepsilon_c \approx -W_c \sim 1/a_z$$

If n : local concentration of the correlated liquid



$$\varepsilon_c(2n) < \varepsilon_c(n)$$

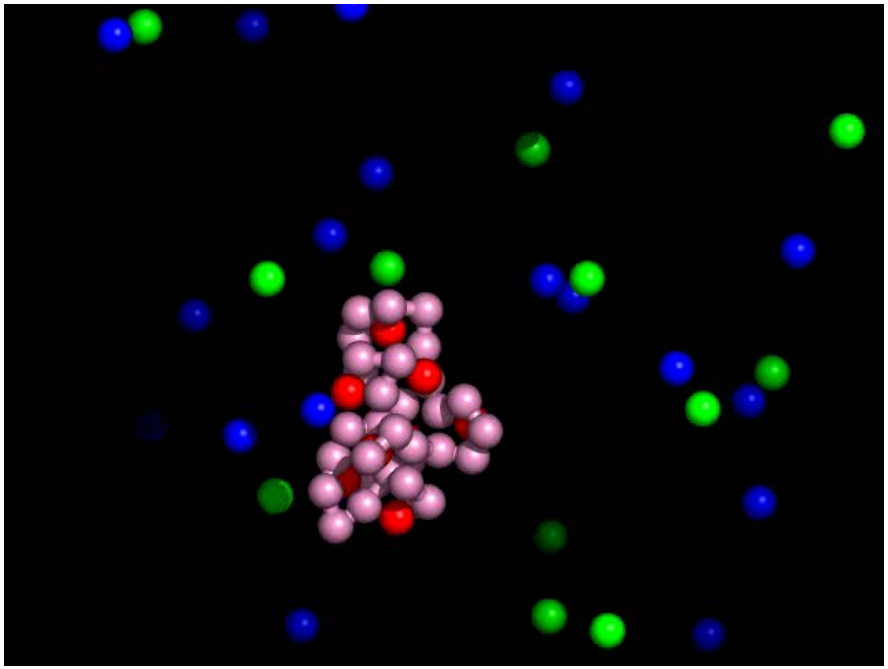
Attraction between DNA

Polyelectrolyte (PE) Chains in multivalent ions (Gonzalez-Mozuelos et al 95;Solis et al 00, 01)

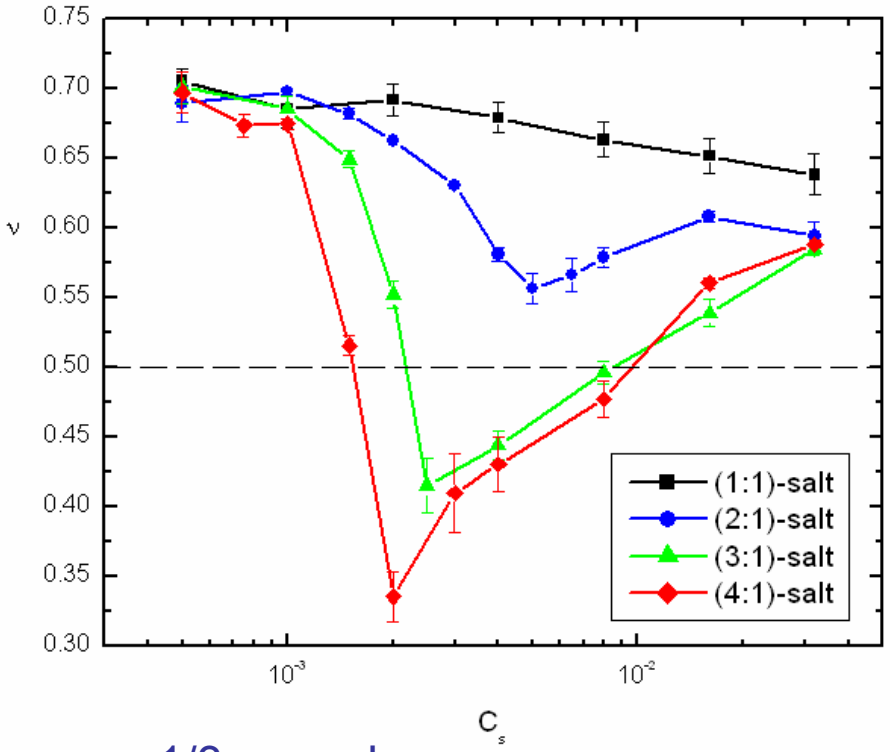
E. Luijten 04

$\sigma_m \approx 2.5$Å; water at 298K: $l_B \approx 7.1$Å $\Rightarrow$ $l_B / \sigma_m \approx 3$; $C_m = 0.008\sigma_m - 3 \sim 1$M

PE(N=32) + 8(4:1)salt



Scaling description: $R_g \sim N^{\nu}$



–1 mer

+1 (+1)

+4 (+4)

–1 coion

Charged rods (DD DNA) the precipitation is into bundles

$\nu = 1/3$ sphere

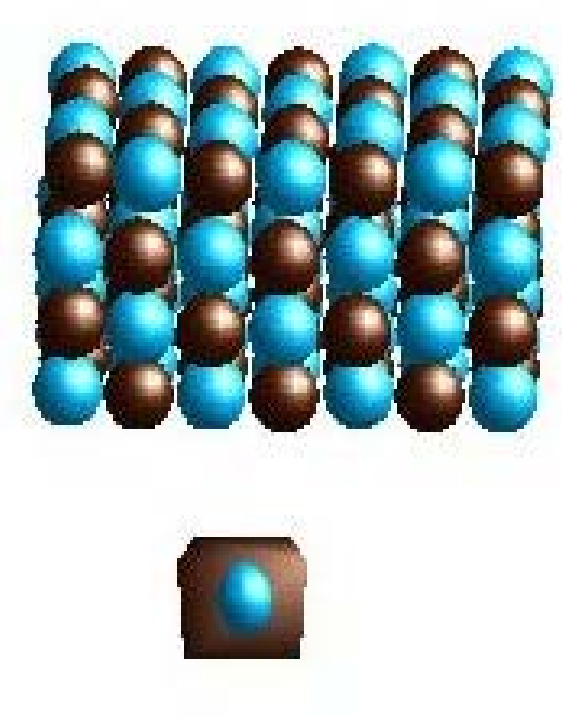
$\nu = 0.5$ ideal chain

$\nu = 0.588$ coil

$\nu = 1$ rigid rod

For collapse chains we use methods from solid state: The Energies of Neutral Dense Systems.

- Fluctuations affect very little the energy of an ionic crystal.
- The basic result is that the sum of energies is convergent and it can be expressed as a cohesive energy per condensed ion e=-M l_B/d.
- l_B is the Bjerrum length, d is the distance between ions and M is the Madelung constant (some use the Wigner argument and calculate the energy of the minimal neutral region).

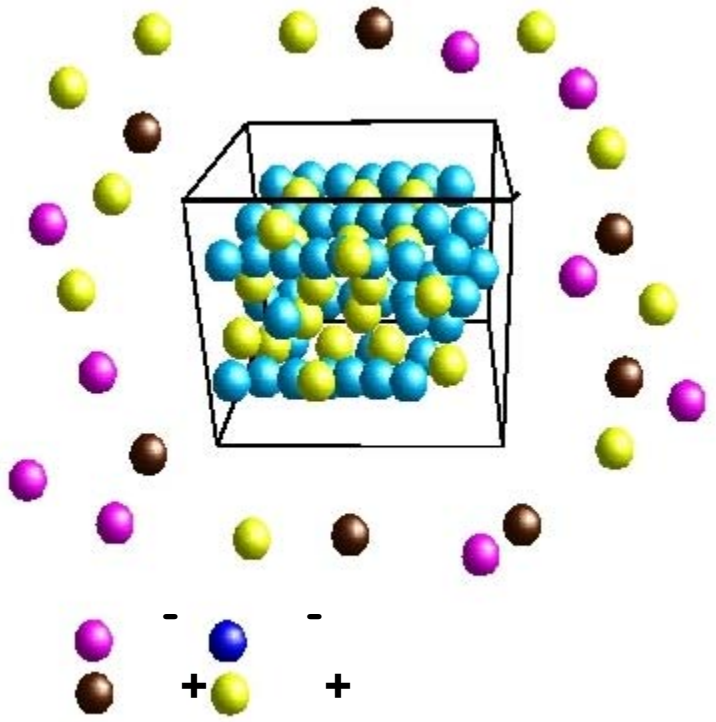


Theoretical idea to explain precipitation and redissolution.

- We separate the charged object from its environment by an imaginary boundary between the dense macroion region and the mean field region.

- The condensed region should be treated by nonperturbative methods.

- We compare free energies of expanded or condensed chains.



We cannot use a single functional to describe
both the liquid and quasi-solid regions.

The radius of gyration changes with amount of salt added.
In high amounts of salt chains are not so stretched.

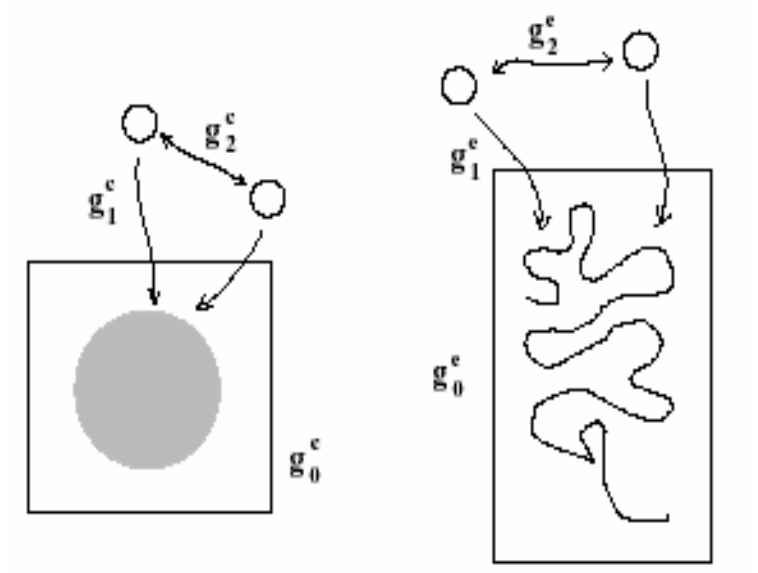
Entropy per chain, however, has little influence.
We can also treat the case of a comparison of rod
to a bundle of rods, and get very similar results

Electrostatic free energies of collapse and expanded chains.

•The electrostatic free energy per monomer per KT is calculated in each case as the sum of three terms,

- $F=g_0+g_1y+g_2y^2$

•In each case the reference state g_0 is neutral.
•The fraction of excess charge condensed is y.



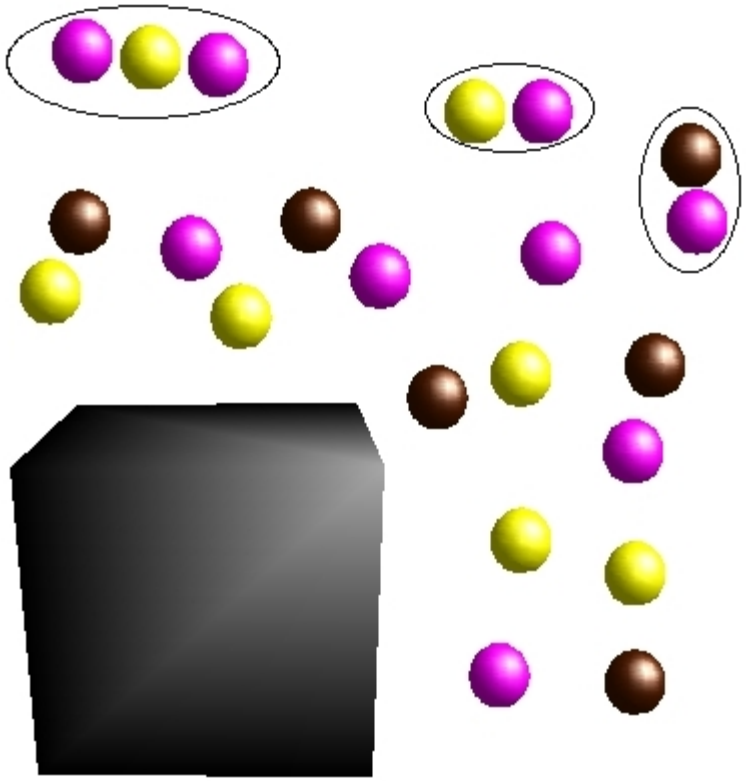
The excess charge y is removed from the solution and we can use

$$F_s = (\partial_y F_s) y = \mu y$$

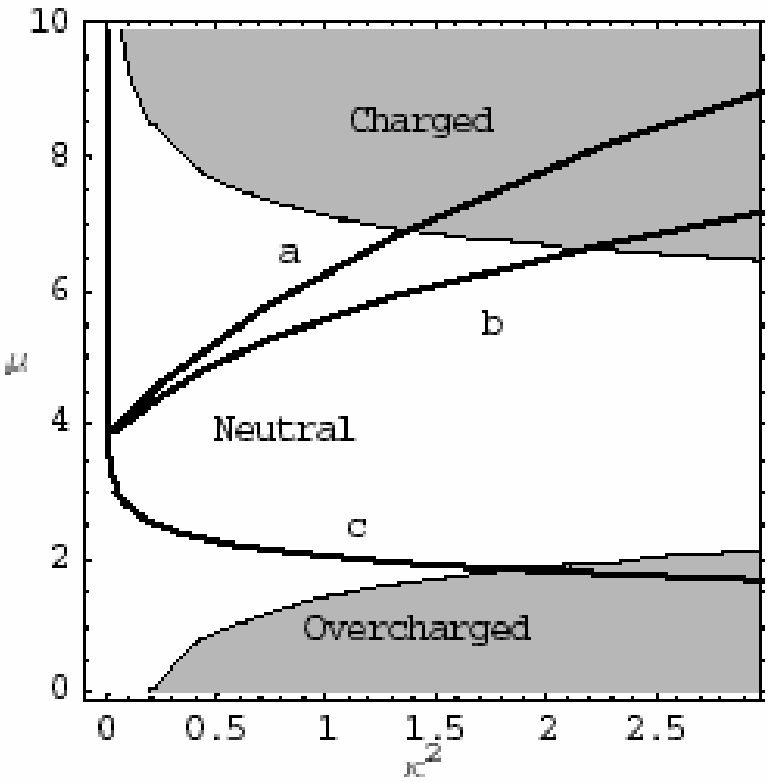
Modeling the energy of the free counterions and solvent.

At low salt the "free" region can be treated by a regular solution model takes care of the entropy of the species, and a Debye-Huckel term accounts for the electrostatic interactions between free particles.

At high salt concentrations, however, is important to also consider ion pairs. Ignoring association leads to unphysical predictions (charge inversion).



$$F_s = (\partial_y F_s) y = \mu y$$



-Precipitation $zm \approx \phi$

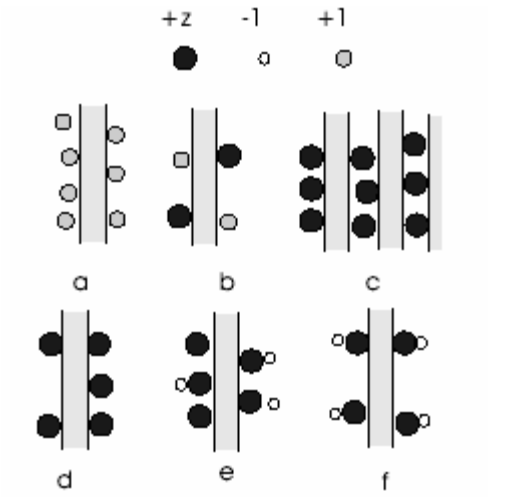
-If B is very large (high cohesive energy of dense structure) then there may not be re-dissolution on the addition of multivalent salt.

a=D-H for point ions,
b=ion-pairs (similar to hard core),
c=only entropy of ions

Debye-Hückel ionic atmosphere and the ionic size effect reduced and may suppressed the DNA overcharging

(Solis & Olvera de la Cruz, 2001)

Effective charge

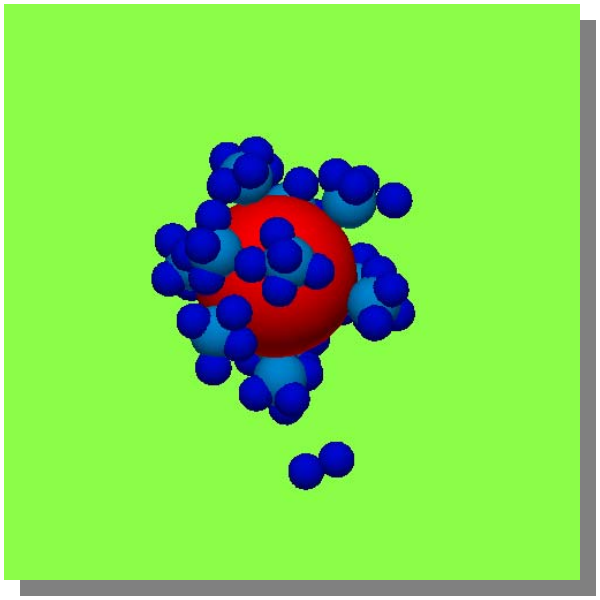


d=positive; e=Neutral, f=negative

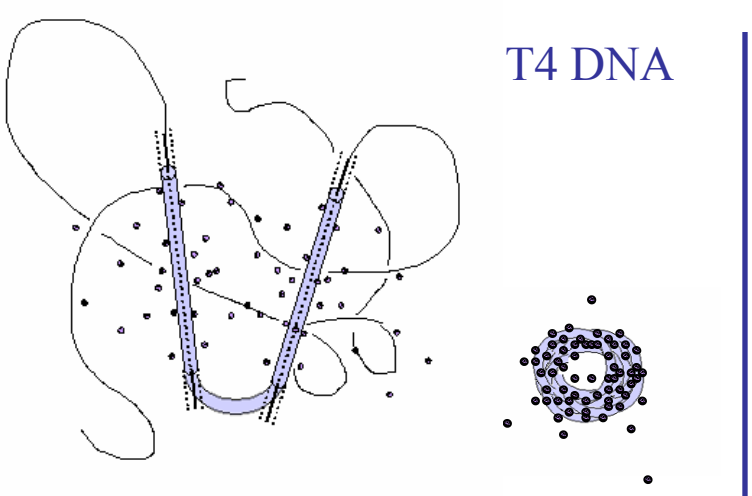
Solis + MO de la Cruz 00, 01; Mesina, Holm and Kremer 00; Solis 02, Grosberg et al 02, Grosberg + Tanaka 01

Stability of complexes to segregation or to dissolution

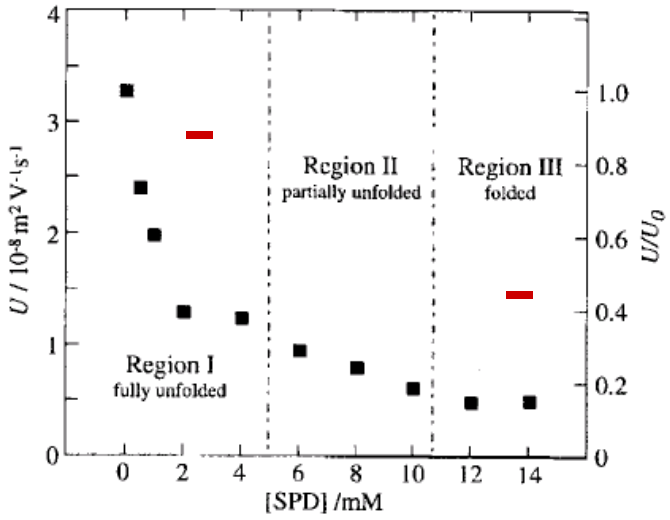
Polyelectrolytes collapse; beyond neutralization, re-expansion occurs because the multivalent ions can interact more with co-ions
Counterion size has crucial role.



About the charge inversion,

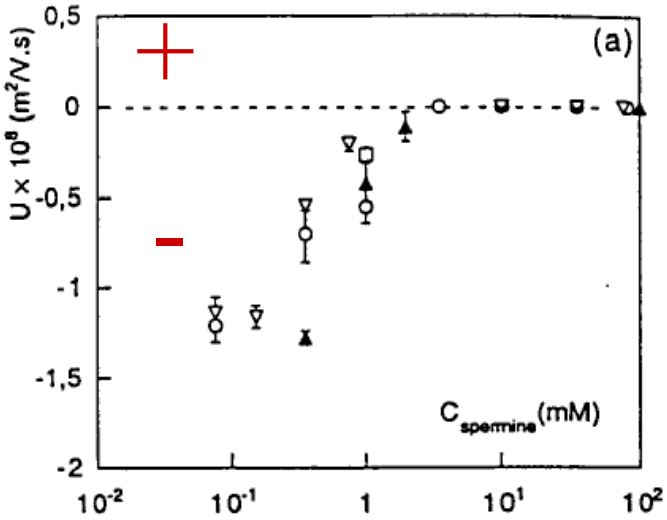


T4 DNA

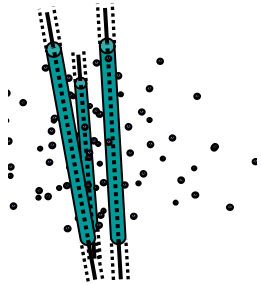


Spermidine concentration ($Z_c = +3$)

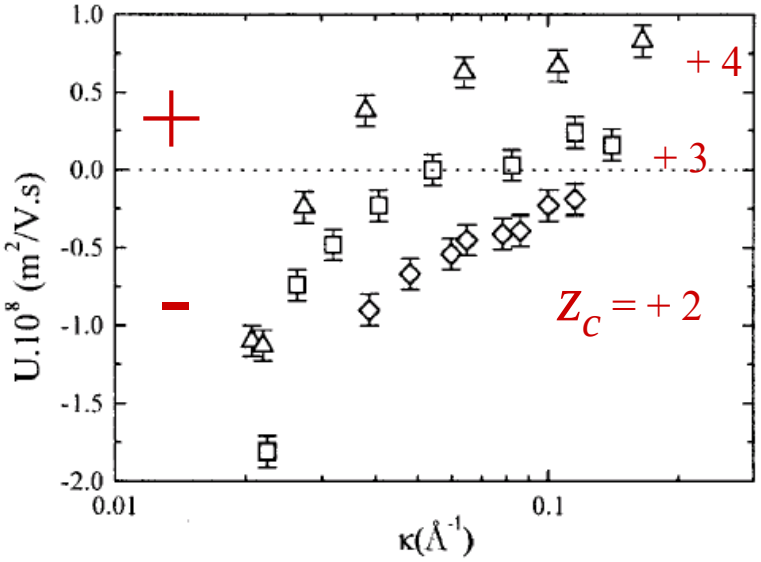
(Yamasaki et al., 2001)



Short DNA fragments



Spermine concentration ($Z_c = +4$)



Nucleosome Core Particles

(Raspaud et al., 1999; De Frutos et al., 2001)

- mixture of mono and z_c-valent cations condensed onto the DNA

attenuated correlation effect

- non-ideal behavior of the “free” multivalent salt

Debye-Hückel ionic atmosphere and the ionic size effect

reduced and may suppressed the DNA overcharging

(Solis & Olvera de la Cruz, 2001)

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